Black holes in chains

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Main claim: We demonstrate that the **Hawking–Page transition** in AdS_5 can be simulated on a 1d **spin chain**.

Based on arXiv:**2401.13963**
"Hawking–Page transition on a spin chain"
with David Pérez-García and Miguel Tierz

The big picture

'*It from quibit*' approach to understand quantum gravity:

- Renewed impulse in the quantum information $\leftrightarrow$ quantum gravity correspondence;
- Recent progress on long-standing problems: Hawking paradox, wormholes, ...;

Quantum simulation envisions devices that reproduce physical phenomena otherwise not directly observable:

- Experimental progress using spin chains and optical lattices;
- Simulation of a wormhole on small quantum computer *[Jafferis et al.]*.

Gravity from qubits

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We provide a proof of concept that the **Black Hole (BH) formation** can be simulated on a simple 1d spin chain.

Black holes in chains

We propose: phase transition for appearance of BH is captured by a variation of Heisenberg's isotropic XY chain.

- We study BH formation $\Longrightarrow$ predominantly thermal event.
- Consistently, we find that simulation requires finite temperature.
- We work in supersymmetric AdS_5 and **assume holographic duality**.
- Signatures of BH physics visible for modest size spin chains. However, we require a thermal state experimentally hard to maintain.

Outline

- Overview of the BH setup
- BH entropy computation
- Spin chain computation
- Extensions and improvements of the result

Review: Black hole entropy and Hawking–Page transition

Hawking–Page transition

We want to study the **Hawking–Page** transition in AdS_5, describing the formation of a BH.

1. Low temperature T, gravitational solution is thermal AdS_5.
2. Increasing T, new solution emerges: BH.
3. Above a threshold T_{HP}, BH solution becomes dominant.

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Bekenstein–Hawking formula: BH entropy proportional to horizon area,

$$S = \frac{A_{\text{horizon}}}{4G_{\text{Newton}}}$$

$\Longrightarrow$ Sudden **jump in entropy** above T_{HP}.

Maximally supersymmetric setup

Holographic setup: take AdS/CFT correspondence for granted.

For gauge group $SU(N)$, holographic dictionary: *[Maldacena]*

$$1/G_{\text{Newton}} \sim N^2.$$

Combining with Bekenstein—Hawking relation $S \propto 1/G_{\text{Newton}}$, predicts:

$$\boxed{S \sim \begin{cases} \mathcal{O}(1) & \text{thermal AdS phase} \\ \mathcal{O}(N^2) & \text{BH phase} \end{cases}} \qquad (\star)$$

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Our aim: Simulate the entropy behaviour ($\star$) on a spin chain.

Supersymmetric black hole entropy

AdS$_5$ and $\mathcal{N} = 4$ SYM

We consider string theory on AdS$_5 \times \mathbb{S}^5 \Longrightarrow$ dual to maximally supersymmetric super-Yang–Mills theory ($\mathcal{N} = 4$ **SYM**). *[Maldacena]*.

This is the most studied and best understood example of AdS/CFT correspondence. It passed uncountable stringent checks.

Advantages:

- It is safe to assume validity of AdS/CFT
- Conformally invariant gauge theory $\Longrightarrow$ constrained observables
- Maximally supersymmetric $\Longrightarrow$ tractable computations

Hawking–Page transition and $\mathcal{N} = 4$ SYM

Hawking–Page transition in $\mathrm{AdS}_5 \longleftrightarrow$ deconfinement in $\mathcal{N} = 4$ SYM.
[Witten]

Consider $\mathcal{N} = 4$ SYM on Euclidean $\mathbb{S}^1_\beta \times \mathbb{S}^3$, with radius of the circle

$$\beta = 1/T.$$

Computable observables:

- **Superconformal index** $\mathcal{I}^{\mathcal{N}=4} = \mathrm{Tr}\ (-1)^{\mathsf{F}}\ e^{-\beta\left\{\mathsf{Q},\mathsf{Q}^\dagger\right\}}$.
- **Thermal partition function** $\mathcal{Z}^{\mathcal{N}=4} = \mathrm{Tr}\ e^{-\beta\left\{\mathsf{Q},\mathsf{Q}^\dagger\right\}}$.

Thermal partition function vs entropy

Combine the statements:

- $\mathcal{Z}^{\mathcal{N}=4}$ is trace over Hilbert space in radial quantization on $\mathbb{S}^3$.
- Holographic duality $\mathbb{S}^1_\beta \times \mathbb{S}^3 \leftrightarrow$ near-horizon geometry of BH in asymptotically AdS_5.
- Thermodynamic relation

$$S = \log(\# \text{ miscrostates}).$$

$\Longrightarrow$ Obtain

$$\boxed{S \propto \ln \mathcal{Z}^{\mathcal{N}=4}_{\mathbb{S}^3\times\mathbb{S}^1}}$$

Thermal partition function vs SCI

Partition functions on Euclidean $\mathbb{S}^1_\beta \times \mathbb{S}^3$

$$\mathcal{I}^{\mathcal{N}=4} \text{ vs } \mathcal{Z}^{\mathcal{N}=4}$$

They differ in the boundary conditions for fermions along $\mathbb{S}^1_\beta$.

- $\mathcal{I}^{\mathcal{N}=4}$ is independent of coupling $\lambda := g^2_{\text{SYM}} N$. *[Witten]*
- $\mathcal{Z}^{\mathcal{N}=4}$ does depend on $\lambda \Longrightarrow$ *a priori* unjustified to compute in weak coupling limit.
- $\mathcal{Z}^{\mathcal{N}=4}$ does not have boson/fermion cancellations $\Longrightarrow$ more sensitive to **deconfinement** transition.

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(i) Take weak coupling limit $\Longrightarrow$ only tree-level gauge invariant constraints, no $\mathcal{O}(\lambda)$ loops;

(ii) EFT approach to compute $\mathcal{Z}^{\mathcal{N}=4}$;

(iii) No phase transition in marginal coupling $\lambda \Longrightarrow$ interpolate to strong coupling.

Caveat: This does not give *exact* entropy, due to $\mathcal{O}(\lambda)$ corrections becoming important in BH phase.

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EFT for the thermal partition function

Study path integral of $\mathcal{N} = 4$ SYM on Euclidean $\mathbb{S}^1_\beta \times \mathbb{S}^3$.

Curvature couplings make all fields massive, except **holonomy** of gauge field around thermal $\mathbb{S}^1_\beta \Longrightarrow$ effective description

- EFT in terms of the only light d.o.f.

$$U = \mathrm{P}\ \exp\left(\oint_{\mathbb{S}^1_\beta} A\right).$$

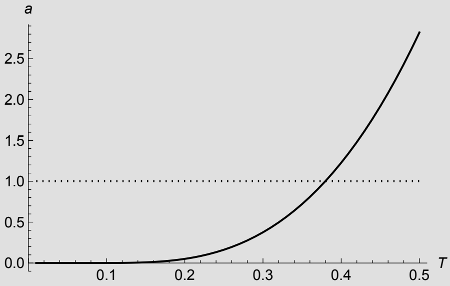
- Gauge field is in the adjoint $\Longrightarrow$ only possible effective interactions

$$a_n\ \frac{1}{n}\ \mathrm{Tr}(U^n)\mathrm{Tr}(U^{-n})$$

Couplings a_n of the effective Lagrangian are obtained integrating out massive modes. *[Sundborg, Aharony et al]*

They depend in complicated way on inverse temperature $\beta = 1/T$. For example,

$$a := a_1 = \frac{2(3e^{1/(2T)} - 1)}{(e^{1/(2T)} - 1)^3}.$$



Entropy from EFT

EFT computation yields *[Sundborg, Aharony et al]*

$$e^S \propto \mathcal{Z}^{\mathcal{N}=4} = \oint \mathrm{d}U \ \exp\left(\sum_{n\geq 1} \frac{a_n}{n} \mathrm{Tr}\left(U^n\right) \mathrm{Tr}\left(U^{-n}\right) \right) .$$

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Simplification: At large N, S is well approximated by the **much simpler** *[Aharony et al x2, Liu]*

$$\boxed{e^{\hat{S}} = \oint \mathrm{d}U \ \exp\left(a \mathrm{Tr}\left(U\right) \mathrm{Tr}\left(U^{-1}\right)\right).} \tag{1}$$

Higher coupling are irrelevant deformations $\Longrightarrow$ unimportant for consideration of **Hawking–Page transition**.

Finding the entropy

Trick: Hubbard–Stratonovich transformation to rewrite

$$\begin{aligned} e^{\hat{S}} &= \int_0^\infty \sigma \mathrm{d}\sigma \oint \mathrm{d}U \ \exp\left[-N^2 \left(\frac{\sigma^2}{4a} - \frac{\sigma}{2N} \mathrm{Tr}\left(U + U^{-1} \right) \right) \right] \\ &= \int_0^\infty \sigma \mathrm{d}\sigma \ \exp\left[-N^2 \left(\frac{\sigma^2}{4a} - \mathcal{F}_{\mathsf{GWW}}(\sigma) \right) \right] \end{aligned}$$

where $\mathcal{F}_{\mathsf{GWW}}$ is the free energy of the Gross–Witten–Wadia matrix model.

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Solve large N limit of $\hat{S}$ in two steps:

(i) Solve GWW integral at large N;

(ii) Saddle point analysis on σ.

Large N entropy

Step (i) is well-known,

$$\mathcal{F}_{\mathrm{GWW}}(\sigma) = \begin{cases} \frac{1}{4}\sigma^2 & \text{if } \sigma \leq 1 \\ \sigma - \frac{1}{2}\ln\sigma - \frac{3}{4} & \text{if } \sigma > 1 \end{cases}$$

Plugging into the integral and extremizing w.r.t. σ one gets *[Liu]*

- If $a < 1$, the saddle is $\sigma = 0$, thus $\hat{S}(a < 1) \sim \mathcal{O}(1)$;
- If $a > 1$, non-trivial saddle at $\sigma > 0$, and $\hat{S}(a > 1) \sim \mathcal{O}(N^2)$.

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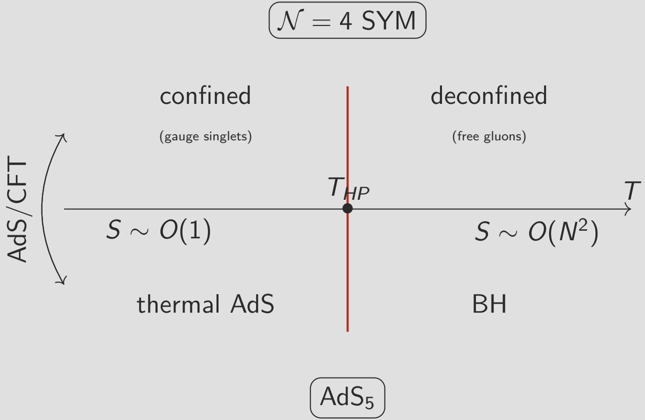
$\Longrightarrow$ **1st order phase transition with behaviour** ($\star$), at

$$T_{HP} \text{ such that } a(T_{HP}) = 1$$

Summary of the entropy calculation



Map to the spin chain

Spin chain setup

Spin-$\frac{1}{2}$ chain of L sites.

- On-site Hilbert space spanned by $|\uparrow\rangle, |\downarrow\rangle$;
- Total Hilbert space is (projectivization of) tensor product over all sites.

Prepare the **initial state** ($L \gg N$)

$$|\psi_0\rangle = |\underbrace{\downarrow\downarrow \dots \downarrow}_{N} \uparrow\uparrow \dots \uparrow\rangle.$$

Temperature of the setup is $\tilde{T} > 0$.

Isotropic Heisenberg chain

Ferromagnetic interaction, $\tilde{J} > 0$, with Hamiltonian

$$H = -\frac{\tilde{J}}{2} \sum_{j=0}^{L-1} \left(\sigma_j^- \sigma_{j+1}^+ + \sigma_j^+ \sigma_{j+1}^- \right), \qquad (2)$$

where $\sigma_j^\pm = \frac{\sigma_j^x \pm i \sigma_j^y}{2}$ are the spin-flip operators on the site j.

They satisfy:

$$\sigma^+ |\downarrow\rangle = |\uparrow\rangle, \quad \sigma^- |\uparrow\rangle = |\downarrow\rangle, \quad \sigma^+ |\uparrow\rangle = 0 = \sigma^- |\downarrow\rangle$$
$$\left[\sigma_j^+, \sigma_k^- \right] = \sigma_j^z \delta_{jk}, \qquad \left[\sigma_j^z, \sigma_k^\pm \right] = \pm 2 \sigma_j^\pm \delta_{jk}.$$

Loschmidt echo

We are interested in **Loschmidt echo** of the spin chain. We consider **thermal** states.

Loschmidt amplitude in the state $|\psi_0\rangle$

$$\mathcal{G}_N(J) = \langle\psi_0| e^{-H/\tilde{T}} |\psi_0\rangle$$

$\Longrightarrow$ the corresponding probability is called Loschmidt echo:

$$\mathcal{L}_N(J) = |\mathcal{G}_N(J)|^2 \, .$$

Remarks on the echo

Remarks on the observable $\mathcal{L}_N(J)$:

- Loschmidt echo (aka fidelity) is return probability to initial state.
- For this H, it only depends on the ratio $J := \tilde{J}/\tilde{T}$.
- It depends on N through the choice of $|\psi_0\rangle$.
- It is not a probability, not correctly normalized to 1. We consider the ratios

$$\hat{\mathcal{L}}_N(J) = \mathcal{L}_N(J)/\mathcal{L}_1(J). \tag{LE}$$

Computing the echo

We want a better rewriting of the amplitude $\mathcal{G}_N(J)$ *[Bogoliubov, PerezGarcia-Tierz]*.

Define $|\Uparrow\rangle \equiv |\uparrow, \uparrow, \dots, \uparrow\rangle$ and

$$g_{j,k}(J) = \langle \Uparrow | \sigma_j^+ e^{-H/\tilde{T}} \sigma_k^- | \Uparrow \rangle.$$

In **thermodynamic limit** $L \to \infty$, N arbitrary, it holds that

$$\langle \Uparrow | \left(\bigotimes_{j=0}^{N-1} \sigma_j^+ \right) e^{-H/\tilde{T}} \left(\bigotimes_{k=0}^{N-1} \sigma_k^- \right) | \Uparrow \rangle = \det_{0 \le j,k \le N-1} \left[g_{j,k} \right].$$

We use this in combination with $|\psi_0\rangle = \bigotimes_{k=0}^{N-1} \sigma_k^- |\Uparrow\rangle$.

Remark: Derivation extends to finite L *[LS-Tierz]*. Thermodynamic limit is enough for today's talk.

Differential equation for the amplitude

We compute

$$\frac{\mathrm{d}g_{j,k}}{\mathrm{d}J} = \frac{1}{2}\langle\Uparrow| \left(\sigma^{+}_{j-1} + \sigma^{+}_{j+1}\right) e^{-H/\tilde{T}} \sigma^{-}_{k} |\Uparrow\rangle$$
$$= \frac{1}{2}\left(g_{j-1,k} + g_{j+1,k}\right)$$

using commutation relations to pass H through the Pauli matrices.
Initial condition is $g_{j,k}(0) \propto \delta_{j,k}$.

Differential equation and echo

$\Longrightarrow$ **Differential eq. for Bessel function** $I_{j-k}(J)$. We find:

$$\frac{\mathcal{G}_N(J)}{\mathcal{G}_1(J)} = \frac{1}{I_0(J)} \det_{0\le j,k\le N-1} \left[I_{j-k}(J) \right] = \frac{1}{I_0(J)} \mathcal{Z}_N^{\mathrm{GWW}} \left(N\sigma = J \right).$$

Second equality is Heine–Szegő identity.

We have obtained that the Loschmidt echo of this spin chain satisfies:

$$\sqrt{\hat{\mathcal{L}}_N(J)} \propto \left| \frac{1}{I_0(J)} \mathcal{Z}_N^{\mathrm{GWW}} \left(N\sigma = J \right) \right|.$$

Relation with BH entropy

We introduce **average over coupling**:

$$\left\langle \sqrt{\hat{\mathcal{L}}_N} \right\rangle_{2a} = \int_0^\infty J \mathrm{d}J \; e^{-\frac{J^2}{4a}} \sqrt{\hat{\mathcal{L}}_N(J)}$$

Comparing with BH entropy formula we get **main result:**

$$\boxed{e^{\hat{S}} \sim \left\langle \sqrt{\hat{\mathcal{L}}_N} \right\rangle_{2a} .}$$

- The dependence on the lab temperature $\tilde{T}$ is averaged out. Free parameter is standard deviation $a > 0$.
- Equality up to normalization factors, unimportant at large N.

Summary of the main claim

We have set up

- A simple spin chain and Hamiltonian;
- A suitable initial state $|\underbrace{\downarrow\downarrow\dots\downarrow}_{N}\uparrow\uparrow\dots\uparrow\rangle$

We have computed the **coupling-averaged Loschmidt echo** (return probability), and shown that it gives the **BH entropy** studied previously.

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- This is **not** a correspondence.
- Free parameter: BH temperature vs standard deviation.
- To produce a jump in the echo that simulates the BH entropy,
  - $\underline{0 < a \ll 1}$: the average is sharply peaked, introducing "a small amount of disorder";
  - $\underline{a \gg 1}$: the average introduces more entropy in the system because no value of J is preferred, resulting in "more disorder".

Refined probes of the transition

More supporting evidence

The computation of BH entropy relies on several **simplifying assumptions** and only gives approximate estimate.

Can we enrich the spin chain setup to simulate more realistic features of the Hawking–Page transition?

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Can we enrich the spin chain setup to simulate more realistic features of the Hawking–Page transition?

Yes! Loschmidt echo = useful device to reproduce the desired features of the transition in a controlled setting.

Perturbative corrections

Beyond tree level in 't Hooft coupling $\lambda \Longrightarrow$ Insert $\mathcal{O}(\lambda)$ corrections.

Corrected BH entropy is given by *[AlvarezGaume-Gomez-Liu-Wadia]*

$$\begin{aligned} e^{\hat{S}_\lambda} &\propto \int_{-\infty}^{\infty} \frac{\mathrm{d}\mu}{\mu}\ e^{-\frac{N^2}{4b}(\mu-a)^2} \int_0^{\infty} \sigma \mathrm{d}\sigma \oint \mathrm{d}U\ \exp\left[-N^2\left(\frac{\sigma^2}{4\mu} - \mathcal{F}_{\mathsf{GWW}}(\sigma)\right)\right] \\ &= \int_{-\infty}^{\infty} \frac{\mathrm{d}\mu}{\mu}\ e^{-\frac{N^2}{4b}(\mu-a)^2}\ e^{\hat{S}}\Big|_{a\mapsto\mu} \end{aligned}$$

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Derivation as above yields:

$$e^{\hat{S}_\lambda} \sim \int_{-\infty}^{\infty} \frac{\mathrm{d}\mu}{\mu}\ e^{-\frac{N^2}{4b}(\mu-a)^2} \left\langle \sqrt{\hat{\mathcal{L}}_N} \right\rangle_{2\mu}.$$

$\Longrightarrow$ Perturbative corrections (**difficult!**) are simulated on the chain by taking **nested averages** (**easy!**)

Improved BH entropy

In passing from S to $\hat{S}$ we discarded irrelevant operators.

Reinsert operators up to order $K \longleftrightarrow$ Spin chain with **generalized Hamiltonian**, interaction beyond nearest neighbour:

$$H_{\mathrm{gen}} = -\frac{1}{2} \sum_{j=0}^{L-1} \sum_{n=1}^{K} \frac{\tilde{J}_n}{n} \left(\sigma_j^- \sigma_{j+n}^+ + \sigma_j^+ \sigma_{j+n}^- \right) ,$$

Procedure as above $\Longrightarrow$ **average** over all spin-spin couplings J_n of the **generalized Loschmidt echo** agrees with the improved estimate of the BH entropy ($K \ll L$).

Fermionization

Fact: The isotropic XY chain is equivalent to free fermions *[Jordan-Wigner]*, mapping

$$|\uparrow\rangle \mapsto |\emptyset\rangle \qquad |\downarrow\rangle \mapsto |1\rangle$$

Fact: This is not true for $K > 1 \implies$ two different fermionic models:

- Straightforward generalization of free fermion model to $K > 1$;
- **Fermionization** of H_{gen}.

They are **inequivalent**.

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They are **inequivalent**. However, their Loschmidt echoes are the same![1]
Non-trivial, relies on specific choice of initial state.

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Polyakov loop

Polyakov loop is an **order operator** for the phase transition.

Insertion of Polyakov loop in the calculation of the entropy $\longleftrightarrow$ insertion of **impurity** in the initial state. Compute return amplitude

$$\langle \psi_0 | e^{-H/\tilde{T}} | \underbrace{\downarrow\downarrow \dots \downarrow}_{N-1} \uparrow, \downarrow, \uparrow \dots \uparrow \rangle$$

in presence of impurity and then average over couplings.

- Perfect agreement, spin chain can incorporate the loop vev.
- Novel statement in spin chains literature: Return amplitude in presence of impurity is **order parameter**.

Complex couplings = Time-evolved echo

Derivation of S holds on a preferred slice of the parameter space *[Sundborg, Aharony et al]*. In general, one allows $a_n \in \mathbb{C}$ *[Copetti-Grassi-Komargodski-Tizzano]*.

Our manipulations hold true, but now we would have $a \in \mathbb{C}$ for standard deviation of **coupling average**. Write $a = |a| e^{\mathrm{i}\varphi}$ and change integration variable $J \mapsto J e^{\mathrm{i}\varphi/2}$ when computing the average.

$\Longrightarrow$ Understand it as real ferromagnetic coupling $\tilde{J} > 0$ at **complex temperature** $T_{\mathbb{C}} = \tilde{T} e^{\mathrm{i}\varphi/2}$. We have:

$$it + \beta := 1/T_{\mathbb{C}}$$

$\Longrightarrow$ reproduce complex parameters of BH entropy by **thermal + time-evolved** Loschmidt echo, at time t and temperature $1/\beta$.

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