Glue-on AdS holography for $T\bar{T}$-deformed CFTs

An extended AdS / $T\bar{T}$ duality *(beyond infinity)*

Wen-Xin Lai 赖文昕 *in collaboration with*
Luis Apolo, Peng-Xiang Hao 郝鹏翔 and Wei Song 宋伟

arXiv:2303.04836

Yau Mathematical Sciences Center,
Tsinghua

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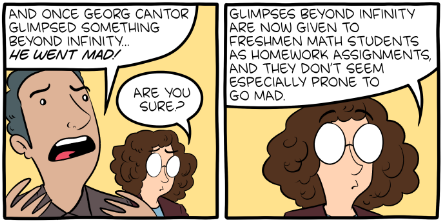

https://smbc-comics.com/comic/cantor

Outline

1. **Warmup:** Pure Einstein gravity in AdS_3 / CFT_2
2. **Review:** $T\bar{T}$ deformation and cutoff AdS
3. **Proposal:** Glue-on AdS holography
4. **Check:** $T\bar{T}$ deformed charges & partition functions
5. **Check:** $T\bar{T}$ partition functions from glue-on AdS
6. Summary & future directions

Warmup: Pure Einstein gravity in AdS_3 / CFT_2

AdS/CFT — holography duality

Maldacena, hep-th/9711200 [2] — "The Large N limit of superconformal field theories and supergravity"

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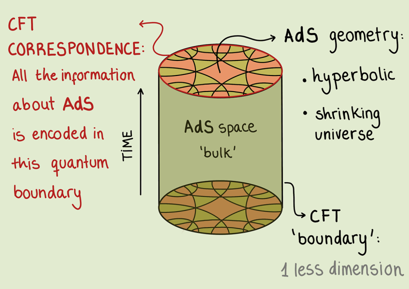

Figure: $\mathrm{AdS}_3/\mathrm{CFT}_2$ cartoon
Image courtesy: *Aldegunde*, 2022 [1]

AdS/CFT — a model of quantum gravity

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- A model of QUANTUM GRAVITY
- ... originating from STRING THEORY
- ... seemingly UNIVERSAL / UBIQUITOUS
  - Model agnostic: $\mathrm{AdS}_5 \times S^5$, $\mathrm{AdS}_3 \times S^3 \times T^4$, symmetric orbifolds, minimal models, "monsters", ... *Maldacena, Witten, GKP, ABJM, Gaberdiel, and many many more*
  - Our work: pure Einstein gravity in AdS_3 / CFT_2 $(d = 2)$.

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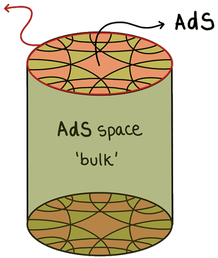
$\mathrm{AdS}_3/\mathrm{CFT}_2$ — gravity trapped in a cylinder

Aharony, Gubser, Maldacena, Ooguri & Oz, hep-th/9905111 [5]



Aldegunde, 2022 [1]

- AdS_3 metric, $\rho \sim z^2 \sim r^{-2}$

$$ds^2 = \ell^2 \frac{d\rho^2}{4\rho^2} + \left(\frac{1}{\rho} g_{ij}^{(0)} + g_{ij}^{(2)} + \rho g_{ij}^{(4)} + \cdots \right) dx^i dx^j \quad (1)$$

boundary at $\rho \to 0$, $x^i \sim \varphi, t$, $i = 1, 2$

- Surprisingly the expansion simply terminates at $\rho g_{ij}^{(4)}$ for pure Einstein gravity! Moreover,

$$g_{ij}^{(4)} = \frac{1}{4} g_{ik}^{(2)} g^{(0)\,kl} g_{lj}^{(2)} \quad (2)$$

Fefferman & Graham, 0710.0919 [3] and *Banados*, hep-th/9901148 [4]

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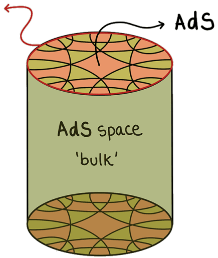

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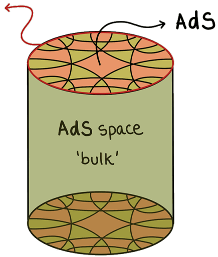
$\mathrm{AdS}_3/\mathrm{CFT}_2$ — simple realization of holography

Banados, Teitelboim & Zanelli, `hep-th/9204099` [6]



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$$ds^2 = \ell^2\left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\,\bar{\mathcal{L}}(v)\,dv\big)\big(dv + \rho\,\mathcal{L}(u)\,du\big)}{\rho}\right)$$

  $g^{(2)} \rightsquigarrow \mathcal{L}(u)$, $\bar{\mathcal{L}}(v)$: arbitrary periodic functions, where $u, v = \varphi \mp it$, $du\,dv = d\varphi^2 + dt^2$

- A consequence of Einstein equations in $(2+1)$ dimensions.
  Too many constraints, too little freedom!

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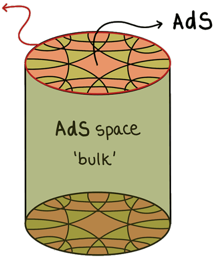

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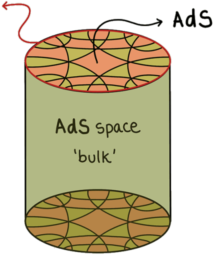

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- Different **geometries** in the "bulk"

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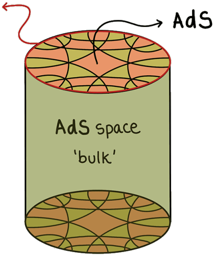
specified by $\mathcal{L}(u)$, $\bar{\mathcal{L}}(v)$: periodic functions,
maps to different **states** in the boundary CFT_2
with stress energy $\langle T(u)\rangle_\psi$, $\langle \bar{T}(v)\rangle_\psi$

- *Dictionary:* bulk $\leftrightarrow$ boundary quantities:
  - spectrum, symmetry,
    partition functions, correlators, ...

Review: $T\bar{T}$ deformation and cutoff AdS

$T\bar{T}$ deformations — motivations

Comments taken from *Cui, Shu, Song & Wang*, 2304.04684 [7]



Aldegunde, 2022 [1]

- Ideally: AdS/CFT
- Reality: non-AdS space and non-CFT
  - possible to extend holographic principle to more general context?
  - attempt: **deformations** on both sides in a controllable way

$T\bar{T}$ deformations — definition

Zamolodchikov, hep-th/0401146 [8], revived by *Smirnov & Zamolodchikov*, 1608.05499 [9] and *Cavaglià, Negro, Szécsényi & Tateo*, 1608.05534 [10] *et al*

- Define "$T\bar{T}$ " as the flow of action:

$$\partial_\mu I = -8\pi \int d^2x\, T\bar{T} = -\pi \int d^2x \left(T^{ij}T_{ij} - (T_i^i)^2\right) \tag{3}$$

For CFT_2, $T\bar{T} = T_{xx}T_{\bar{x}\bar{x}}$, where $x, \bar{x} = \varphi' \mp it'$.

- The stress tensor $T_{ij}(\mu)$ on the right hand side flows with the deformation parametrized by μ.
- This is thus a *differential equation* for a flow, starting from some generic QFT specified by $I(\mu = 0)$. In this work we shall start from CFT_2, but note that $T\bar{T}$ is *irrelevant*: conformal symmetry will be broken.

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"Solvable" deformations on the field theory side

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- **"Solvable":** the deformed spectrum of $\hat{H}(\mu)$ and $\hat{J}(\mu)$ on a cylinder of radius R can be solved, and it's a simple function of the undeformed $E(0)$, $J(0)$:

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu}{R}E(0) + \frac{4\mu^2}{R^4}J(0)^2}\right), \qquad J(\mu) = J(0) \quad (4)$$

  under simple (and reasonable) assumptions (e.g. translation invariance et al).

- **Variants:** suppose the theory has some "components" labeled by w,
  - double-trace $T\bar{T} = (\sum_w T_w)(\sum_w \bar{T}_w)$ (this work)
  - single-trace $T\bar{T}' = \sum_w (T_w \bar{T}_w)$ (e.g. *Cui, Shu, Song & Wang*, 23)

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  - "asymptotic fragility": *Dubovsky, Gorbenko & Mirbabayi*, 1706.06604 [13]
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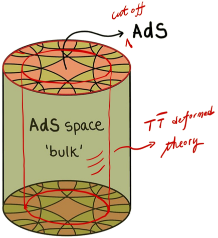
McGough, Mezei & Verlinde, 1611.03470 [17] – "Moving the CFT into the bulk with $T\bar{T}$"



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- Question: what is the AdS dual of $T\bar{T}$ deformation?
  - look up *double-trace deformations* in *the dictionary*: *Heemskerk & Polchinski*, 1010.1264 [14]
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- Answer: AdS gravity within a finite Dirichlet wall

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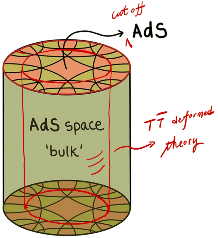

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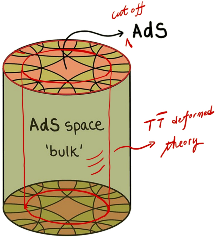

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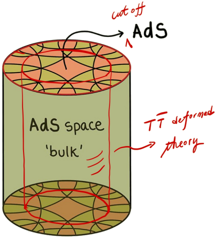

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- Location (radius) of the cutoff surface:

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \tag{6}$$

gets mapped to the deformation parameter μ:

$$\zeta_c = -\frac{c\mu}{3\ell^2} \tag{7}$$

Roughly $\mu \sim 1/r^2$

- Passed many non-trivial tests
- Related to "mixed boundary conditions" by *Guica & Monten*, 1906.11251 [18]

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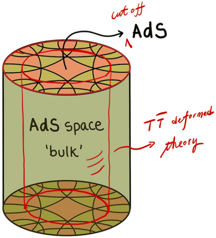

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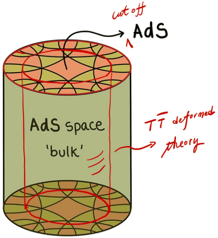

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- **Significance:** gravity inside a finite box with hard Dirichlet walls
  - Dual to some "solvable" no-longer-conformal ft
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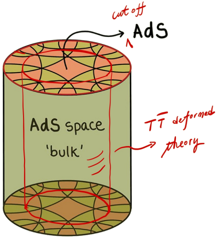

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Proposal: Glue-on AdS holography

Glue-on AdS_3 — analytic continuation

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

- AdS_3 metric has only simple poles with the ρ coordinate:

$$ds^2 = \ell^2\left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\,\bar{\mathcal{L}}(v)\,dv\big)\big(dv + \rho\,\mathcal{L}(u)\,du\big)}{\rho}\right) \tag{9}$$

  thus it admits well-defined analytic continuation from $\rho > 0$ to $\rho \in \mathbb{R}$.

- Continuation of the dictionary:

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \in \mathbb{R}, \qquad \zeta_c = -\frac{c\mu}{3\ell^2} \in \mathbb{R} \tag{10}$$

Glue-on AdS_3 / $T\bar{T}$— updating the dictionary

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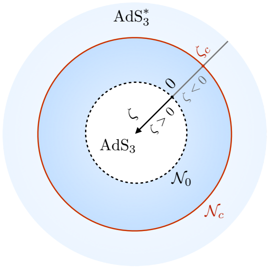

Figure: Glue-on AdS_3

Top-down view of the Poincaré disk

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \in \mathbb{R}, \qquad \zeta_c = -\frac{c\mu}{3\ell^2} \in \mathbb{R} \qquad (11)$$

- What have we done other than copy-pasting?
  - Although the continuation is straightforward, physical quantities diverge as $\rho \to 0$
  - A prescription is required to "renormalize" the divergences

Glue-on AdS_3 / $T\bar{T}$— updating the dictionary

Kraus, Liu & Marolf, 1801.02714 [11] – "Cutoff AdS_3 versus the $T\overline{T}$ deformation"

- Firstly, matching energy momentum (Brown-York) & the flow equations:

$$T_{ij} = \frac{\sigma_{\zeta_c}}{8\pi G}\left(K_{ij} - K\gamma_{ij} + \frac{1}{\ell|\zeta_c|}\gamma_{ij}\right), \qquad \sigma_{\zeta_c} = \frac{|\zeta_c|}{\zeta_c} = \pm 1, \qquad (12)$$

The field theory metric $\gamma_{ij} = \zeta_c h_{ij}$, where h_{ij} is the induced metric.

  - γ_{ij} is always positive-definite, while h_{ij} becomes negative-definite for the glue-on region. This discrepancy is the origin of the $|\zeta_c|$.

- $T\bar{T}$ flow is recast geometrically as the i, j components of the Einstein equations. Note that the geometry can be decomposed such that:

$$ds^2 = \frac{1}{\zeta}\gamma_{ij}dx^i dx^j + n_\mu n_\nu dx^\mu dx^\nu \qquad (13)$$

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Glue-on AdS_3 — beyond the infinity

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

- The geometry is foliated by constant ζ surfaces $\mathcal{N}_\zeta$:

$$\begin{aligned} ds^2 &= \ell^2\left(\frac{d\rho^2}{4\rho^2} + \frac{\left(du + \rho\,\bar{\mathcal{L}}(v)\,dv\right)\left(dv + \rho\,\mathcal{L}(u)\,du\right)}{\rho}\right) \\ &= \frac{1}{\zeta}\gamma_{ij}dx^i dx^j + n_\mu n_\nu dx^\mu dx^\nu, \qquad \frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \end{aligned} \tag{14}$$

  The $T\bar{T}$ deformed theory lives on $\mathcal{N}_{\zeta_c}$ with metric γ_{ij}.

- Treatment for the singularity at $\zeta \to 0^\pm$: introduce $\mathcal{N}_{\zeta=\pm\epsilon}$ and glue them together ("glue-on"); exclude the $-\epsilon < \zeta < \epsilon$ region until finally sending $\epsilon \to 0$.

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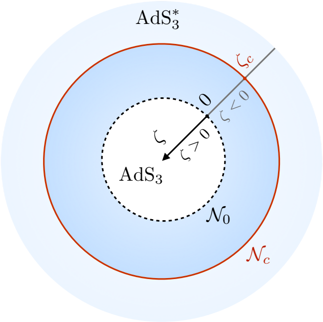

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Check: $T\bar{T}$ deformed charges & partition functions

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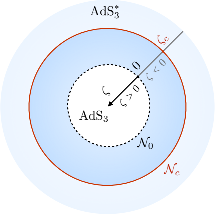

Figure: Glue-on AdS_3

Top-down view of the Poincaré disk

- AdS/CFT: weak / strong duality
  - *quantum* quantities on the CFT side can be computed by (semi-)classical geometry
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- Demanding the extended geometry to be non-singular re-produces bounds on the $T\bar{T}$ deformed theory, e.g.

$$\zeta_c \geq -1 \quad \Leftrightarrow \quad \mu \leq \frac{3\ell^2}{c} \qquad (15)$$

$\zeta_c = -1$ is a horizon of the glue-on geometry, where $\det g_{\mu\nu} = 0$.

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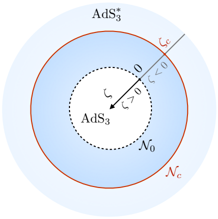

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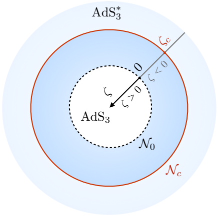

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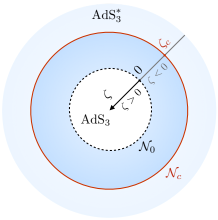

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$$ds_c^2 = \ell^2(-dt'^2 + d\varphi'^2), \qquad \varphi' \sim \varphi' + 2\pi. \qquad (16)$$

- This is realized by the *state-dependent* map of coordinates

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$$T_L(\mu)\,T_R(\mu) \le -\frac{1}{4\pi^2\ell^2\zeta_c} = \frac{3}{4\pi^2 c\mu} = T_H(\mu)^2, \quad \mu > 0 \qquad (17)$$

$T_{L,R}$ are temperatures associated with $u', v' = \varphi' \pm t'$.

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$T\bar{T}$ partition functions from bulk on-shell actions

Caputa, Datta, Jiang & Kraus, 2011.04664 [25]

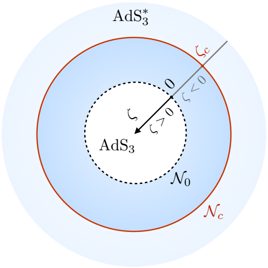

Figure: Glue-on AdS_3

Top-down view of the Poincaré disk

- Bulk: weakly coupled gravity, the partition function is approximated by the on-shell gravity action of the dominant saddle:

$$Z_{T\bar{T}}(\mu) = \mathcal{Z}(\zeta_c) \approx e^{-I(\zeta_c)} \qquad (18)$$

- $I(\zeta_c)$ is roughly the volume of the geometry, which diverges at $\zeta \to 0^{\pm}$. We apply a natural extension of *holographic renormalization* to remove the divergence.

- $I = -\log \mathcal{Z}$ satisfies the $T\bar{T}$ flow (3). This is non-trivial because we've packaged the quantum corrections of the boundary theory.

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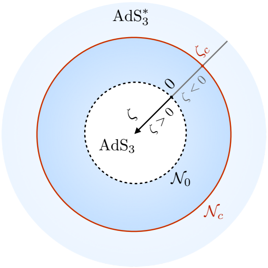

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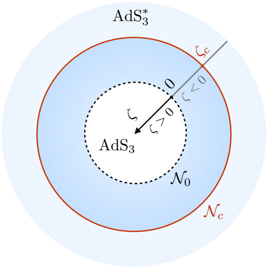

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Comparison with field theory analysis

Datta & Jiang, 1806.07426 [26] and *Apolo, Song & Yu*, 2301.04153 [27]

- **Torus:** Large c, *modular invariance* & sparseness of the "light" spectrum [26, 27], c.f. *Hartman, Keller & Stoica*, 1405.5137 [28]

$$\log Z_{T\bar{T}}(\mu) \approx \begin{cases} -\frac{1}{2}\left(\beta_L + \beta_R\right) RE_{\text{vac}}(\mu), & \beta_L\beta_R > 1, \\ -2\pi^2\left(\frac{1}{\beta_L} + \frac{1}{\beta_R}\right) RE_{\text{vac}}\left(\frac{4\pi^2}{\beta_L\beta_R}\mu\right), & \beta_L\beta_R < 1, \end{cases}$$

- **Sphere:** maximally symmetric, *Donnelly & Shyam*, 1806.07444 [29]

$$\langle T_{ij}\rangle = -\frac{1}{4\pi\mu}\left(1 - \sqrt{1 - \frac{c\mu}{3L^2}}\right)\gamma_{ij}. \tag{19}$$

Sphere partition functions

Donnelly & Shyam, 1806.07444 [29] and *Li*, 2012.14414 [30]

- Given the explicit stress tensor $\langle T_{ij}\rangle \propto \gamma_{ij}$, the flow equations:

$$\partial_\mu \log Z_{T\bar{T}}(\mu) = 8\pi \int d^2x \sqrt{\gamma}\, \langle T\bar{T}\rangle$$

$$-L\,\partial_L \log Z_{T\bar{T}}(\mu) = \int d^2x \sqrt{\gamma}\, \langle T^i_i\rangle$$

- ... admit the general solution with a μ-*independent* integration a:

$$\log Z_{T\bar{T}}(\mu, a) = \frac{c}{3} \log\left[\frac{L}{a}\left(1 + \sqrt{1 - \frac{c\mu}{3L^2}}\right)\right] - \frac{L^2}{\mu}\sqrt{1 - \frac{c\mu}{3L^2}} + \frac{L^2}{\mu}$$

- *Donnelly & Shyam*, 1806.07444 [29] is recovered with $a = \sqrt{c|\mu|/3}$ but one can also take $a = \epsilon$, thus decoupling the energy (RG) scale ϵ with the deformation μ.

Gravitational actions: with counterterms

Donnelly & Shyam, 1806.07444 [29] and *Li*, 2012.14414 [30]

- Gravitational actions, with counterterms:

$$I_{\mathcal{M}}(\zeta_1,\zeta_2) := -\frac{1}{16\pi G}\int_{\zeta_1}^{\zeta_2} d\zeta \int d^2x\sqrt{g}\,(R+2\ell^{-2}),$$

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... consistent with the Brown-York stress tensor T_{ij}, fully renormalized at $\zeta \to 0^{\pm}$.

- The lack of diff invariance of the $\log|\zeta|$ counterterm is known to be a reflection of the Weyl anomaly: *Henningson & Skenderis, de Haro, Solodukhin & Skenderis, Papadimitriou*.

Gravitational actions: with counterterms

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- The $\log$ counterterm makes a crucial contribution to the on-shell action:

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  It guarantees that the partition function is compatible with the $T\bar{T}$ differential equation — not the case for *Donnelly & Shyam*.

- In general the space of deformed theories are parametrized by (μ, a), where a is the length scale (inverse energy scale). In many contexts they are naturally related: $a = \sqrt{c|\mu|/3}$, but a can be tuned by RG.

Summary & future directions

Glue-on AdS_3 / $T\bar{T}$ deformed CFT_2

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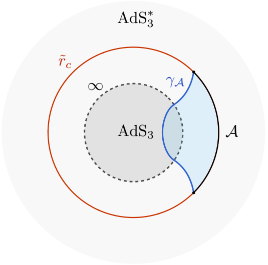

Figure: RT for Glue-on AdS_3

Top-down view of the Poincaré disk

- Holographic proposal for $T\bar{T}$-deformed CFTs with $\mu \in \mathbb{R}$.
- As evidence, we show that the $T\bar{T}$ trace flow equation, the spectrum on the cylinder, and the partition function on the torus and the sphere, among other results, can all be reproduced from bulk calculations in glue-on AdS_3.
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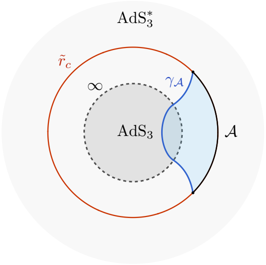

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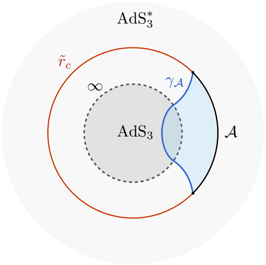

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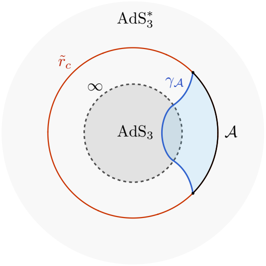

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- We hope to understand the entanglement structure of $T\bar{T}$ deformed theories from bulk geometry.
  - *Donnelly & Shyam*, 1806.07444 [29] and *Lewkowycz, Liu, Silverstein & Torroba*, 1909.13808 [34]
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Thank you!

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