Complex saddles of three-dimensional de Sitter gravity via holography

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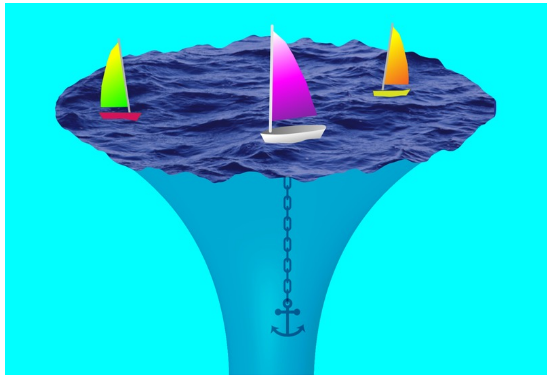
Refs.
[1] YH-Nishioka(Osaka)-Takayanagi(YITP)-Taki(YITP), PRL129(2022)041601
[2] YH-Nishioka-Takayanagi-Taki, JHEP05(2022)129
[3] Chen(NTU)-YH-Taki-Uetoko(Kushiro), arXiv:2302.09219 (+ in preparation)
(cf. Chen-Chen(NTU))-YH, PRL129(2022)061601; JHEP02(2023)038)



From viewpoint in “Physics” (APS)

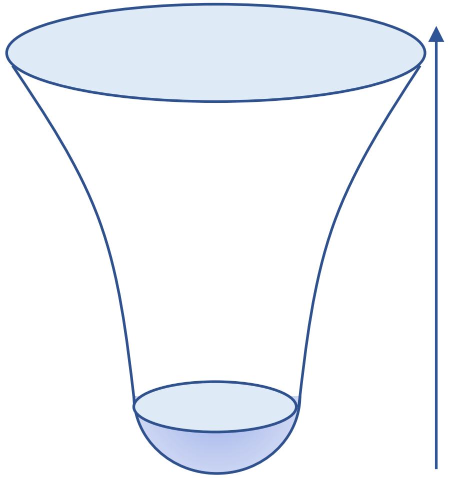
April 19th, 2023@BIMSA, China (online)

Complex geometry

- Complex geometry
  - It is sometimes useful to think of complex geometry in order to quantize gravity theory
- No-boundary proposal by Hartle and Hawking
  - A canonical example of complex geometry
  - Consider a complexifed metric of S^{d+1}

$$ds^2 = L^2(\theta'(u)^2 du^2 + \cos^2\theta(u) d\Omega_d^2)$$

  - S^{d+1}: $\theta(u) = u$, dS_{d+1}: $\theta(u) = iu$
  - Geometry of Hartle-Hawking is realized as a real slice



Allowable complex geometry

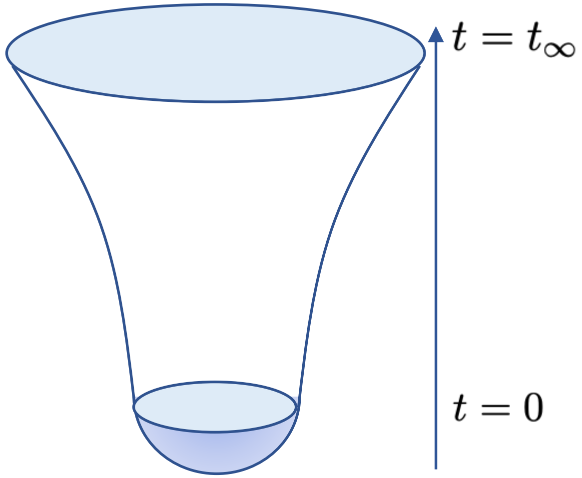
[Louko-Sorkin CQG '97;Kontsevich-Segal QJM '21;Witten'21]

- A complexified metric of S^{d+1}

$$ds^2 = L^2(\theta'(u)^2 du^2 + \cos^2\theta(u) d\Omega_d^2)$$

  - Let us assume that the universe starts from nothing at $u = 0$ and approaches to dS_{d+1} for $u \to \infty$
  - There is a family of complex geometry labeled by *n*

$$\cos\theta(u=0) = 0 \longrightarrow \theta = (n + 1/2)\pi \ (n \in \mathbb{Z})$$



- A criteria of *D*-dim. allowable geometry is

$$\mathrm{Re}\left(\sqrt{\det g} g^{i_1 j_1} \dots g^{i_q i_q} F_{i_1 \dots i_q} F_{j_1 \dots j_q}\right) > 0, \ 0 \le q \le D$$

→ Only geometry with *n*=-1,0 are allowable, which reproduces the geometry of Hartle-Hawking

Allowable geometry via holography

- We determine allowable complex geometry via dS/CFT
  - dS/CFT has not been understood yet compared with AdS/CFT understood as very few concrete examples are available
- A concrete example is given by higher-spin holography

Higher-spin gravity on dS_4 at the classical limit	⟺	3d Sp(N) vector model at the large N limit

[Anninos-Hartman-Strominger ’11 (CQG ’17)]

  - Analytic continuation of duality between higher-spin gravity on AdS_4 and 3d O(N) vector model [Klebanov-Polyakov PLB ’02]

The aim of this talk

1. Propose a dS/CFT correspondence involving 3d higher-spin gravity

Higher-spin dS_3 gravity ($\simeq SL(N)$ CS theory) at the classical limit (+ matters)	⟺	2d $SU(N)$ WZW model with large central charge (+ extra sectors)

2. Provide evidence
   - We show the match of partition functions
   - We relate our proposal to AdS_3 higher-spin holography via analytic continuation [Gaberdiel-Gopakumar PRD '11]
3. Determine allowable geometry
   - Applying the proposed holography, we determine allowable complex geometry as saddle of 3d Chern-Simons gravity from dual 2d CFT

The plan of this talk

- Introduction
- dS_3/CFT_2 and partition functions
- Relation to Gaberdiel-Gopakumar duality
- Complex saddles of Chern-Simons gravity
- Conclusion

dS_3/CFT_2 and partition functions

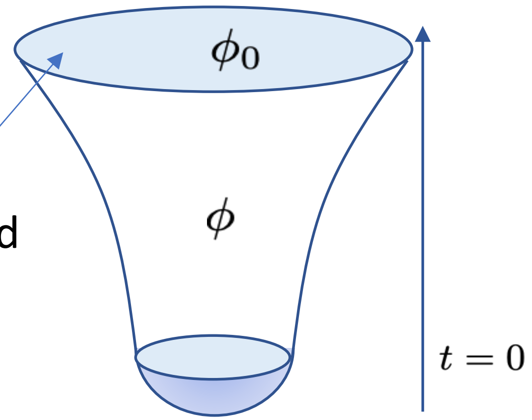
Basics of dS/CFT

[Maldacena JHEP ’03]

- A way to describe gravity theory on dS space is utilized **wave functional of universe**

$$\Psi_{\text{dS}}[h,\phi_0] = \int \mathcal{D}g\mathcal{D}\phi \exp iS[g,\phi]$$

$$\text{with } g = h, \phi = \phi_0 \text{ at } t = t_\infty$$



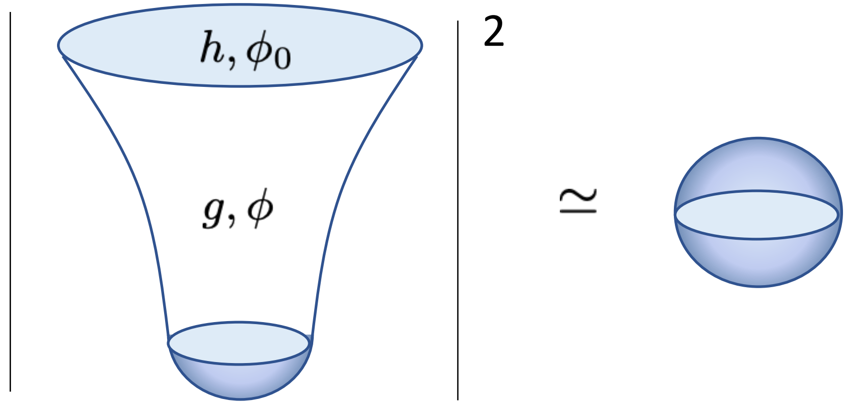
Correlators are computed by dual Euclidean CFT

- The wave functional is identified as generating functional of **correlation functions** in dual CFT

$$\Psi_{\text{dS}}[\phi_0] = \left\langle \exp\left(\int d^d x \phi_0(x)\mathcal{O}(x)\right)\right\rangle$$

- In particular, gravity partition function is computed from square of wave functional

$$Z_{\text{G}} = \int \mathcal{D}h\mathcal{D}\phi_0 |\Psi_{\text{dS}}[h,\phi_0]|^2$$



Our proposal and match of partition functions

- Our proposal

Higher-spin dS_3 gravity ($\simeq SL(N)$ CS theory) at the classical limit (+ matters)	⟷	2d $SU(N)$ WZW model with large central charge (+ extra sectors)

  - In the following we focus on the simplest case with $N=2$

- The match of partition functions
  - We compute the partition functions of 2d SU(2) WZW model on S^2 and find agreement with gravity partition functions
  - We utilize Witten's method to compute the CFT partition functions on S^2 [Witten CMP '89]

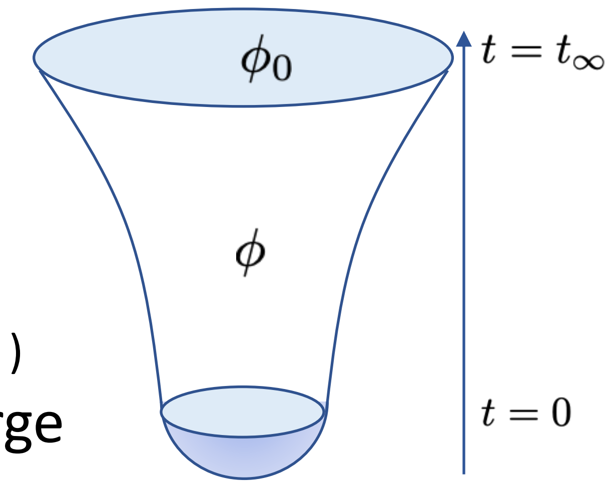
Central charge and the level of WZW model

- Virasoro symmetry appears near the future infinity with central charge [Strominger JHEP '01]

$$c = i\frac{3L_{\mathrm{dS}}}{2G_N} \equiv ic^{(g)}$$

Radius of de Sitter space

Newton constant (classical limit $G_N \to 0$)



  - Dual CFT is quite strange as it has pure imaginary central charge

- 2d SU(2) WZW model with level *k*

$$S = \frac{k}{2\pi}\int d^2z[g^{-1}dg \cdot g^{-1}dg] + k\,\Gamma_{\mathrm{WZ}}, \;\; g \in \mathrm{SU}(2) \;\; \text{with} \;\; c = \frac{3k}{k+2}$$

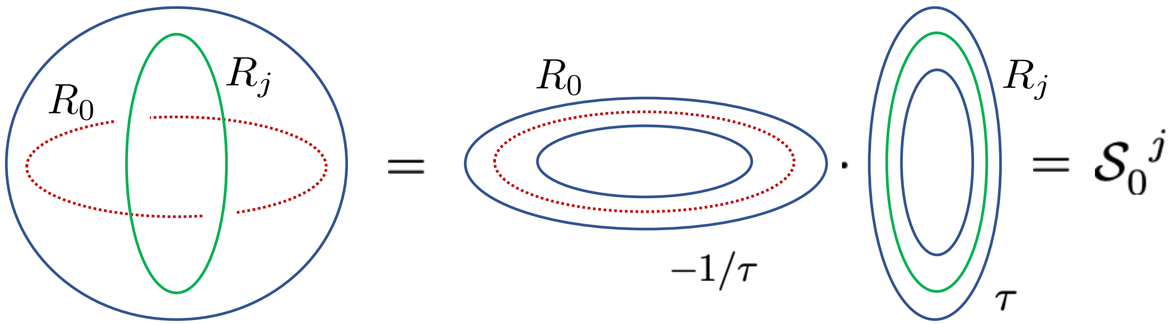
  - To reproduce the requirement from dual gravity, we consider a peculiar limit

$$k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2})$$

Sketch of Witten's method

[Witten'89]

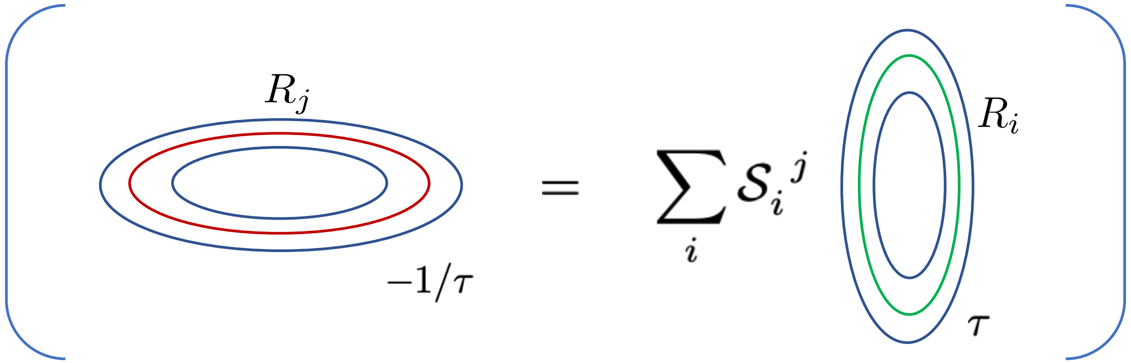
- The partition function of 2d SU(2) WZW model on S^2 can be mapped to that of 3d SU(2) Chern-Simons theory on S^3
  - Wilson loop in spin-*j* rep can be inserted R_j



- Sphere can be obtained by gluing two tori related by *S*-transformation via surgery

$$S : \tau \to -1/\tau$$

  - Modular *S*-matrix of 2d SU(2) WZW model relates the two amplitudes



Partition function on 3-sphere

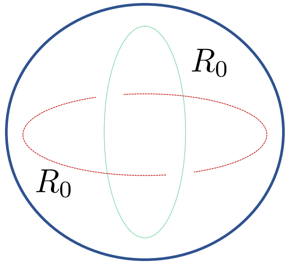
[YH-Nishioka-Takayanagi-Taki'21;'22]

- CFT computation
  - Modular *S*-matrix of 2d SU(2) WZW model

$$\mathcal{S}_j^{\ l} = \sqrt{\frac{2}{k+2}} \sin\left[\frac{\pi}{k+2}(2j+1)(2l+1)\right]$$

  - Vacuum partition function at the leading order in $1/c^{(g)}$

$$\left(k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2}) \right)$$

$$\left| \vphantom{\int} \right.$$$$\left. \vphantom{\int} \right|^2 = |\mathcal{S}_0^{\ 0}|^2 \simeq e^{\frac{\pi c^{(g)}}{3}}$$

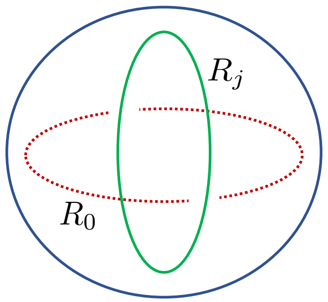
- Gravity computation
  - Definition of classical partition function

$$Z_{\rm G} = e^{-I_{\rm G}}, \ \ I_{\rm G} = -\frac{1}{16\pi G_N}\int d^3x\sqrt{g}(R - 2L_{\rm dS}^{-2})$$

  - Classical action on $\mathbb{S}^3$: $I_{\rm G} = -\frac{\pi L_{\rm dS}}{2G_N} = -\frac{\pi c^{(g)}}{3}$ ⟵ Reproduces CFT computation!!

Wilson loop and bulk excitation

- Partition function on S³ with Wilson loop in rep. R_j



$$= \mathcal{S}_0^{\ j}$$

  - The Wilson line on S³ ⇔ Operator in 2d WZW model [Witten CMP '89]
  - Conformal dimension of CFT operator

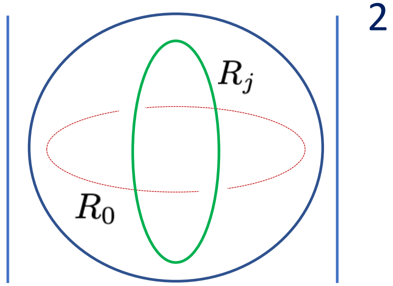
$$\Delta_j = \frac{2j(j+1)}{k+2} \equiv i\Delta_j^{(g)}$$

- dS/CFT map to bulk excitation energy

$$\Delta_j^{(g)} = L_{\mathrm{dS}} E_j$$

Partition function on Euclidean dS_3 black hole

- CFT computation
  - The modular *S*-matrix leads at the leading order in $1/c^{(g)}$



$$= |\mathcal{S}_0{}^j|^2 \simeq e^{\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}}$$

- Gravity computation
  - Bulk excitation creates Euclidean dS_3 black hole (conical defect)

$$ds^2 = L_{\mathrm{dS}}^2\left[(1-8G_N E_j - r^2)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right]$$

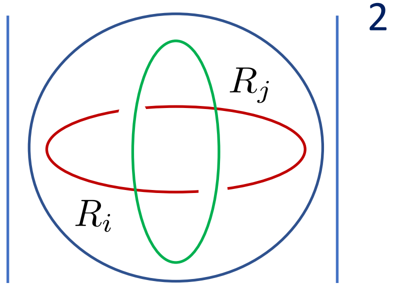
  - Classical action on the geometry

$$I_{\mathrm{G}} = -\frac{\pi}{3}c^{(g)}\sqrt{1-8G_N E_j}$$

Reproduces CFT computation!!

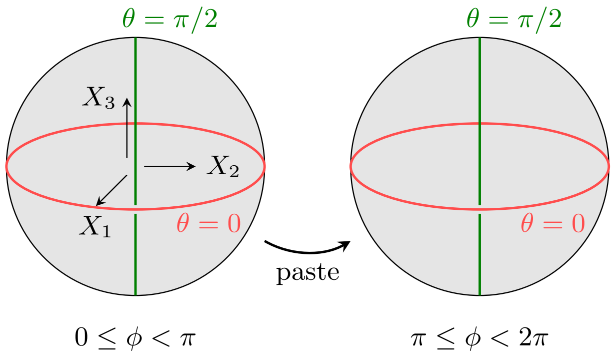
Geometry related to two linked Wilson loops

- CFT computation
  - The modular *S*-matrix leads at the leading order in $1/c^{(g)}$



$$= |\mathcal{S}_i{}^j|^2 \simeq e^{\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_i}\sqrt{1-8G_N E_j}}$$

- Gravity computation
  - Corresponding geometry has two non-trivially linked conical defects



  - Classical action on the geometry

$$I_{\mathrm{G}} = -\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_i}\sqrt{1-8G_N E_j}$$

⟸ Reproduces CFT computation!!

Relation to Gaberdiel-Gopakumar duality

Gaberdiel-Gopakumar duality for AdS_3

[Castro-Gopakumar-Gutperle-Raeymaekers JHEP ’12; Gaberdiel-Gopakumar JHEP ’12]
(see [Gaberdiel-Gopakumar PRD ’11] for original proposal)

- A version of Gaberdiel-Gopakumar duality

Higher-spin AdS_3 gravity (≃SL(N) CS theory) with matters at the classical limit	⟺	2d coset model with large central charge $\frac{\mathrm{SU}(N)_k \times \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$

Spins of gauge fields

$$s = 2, 3, \ldots, N$$

The coset describes analytic continuation of Virasoro-minimal model, which was shown to reduce to Liouville theory [Creutzig-YH JHEP ’21]

- The simplest case with N=2

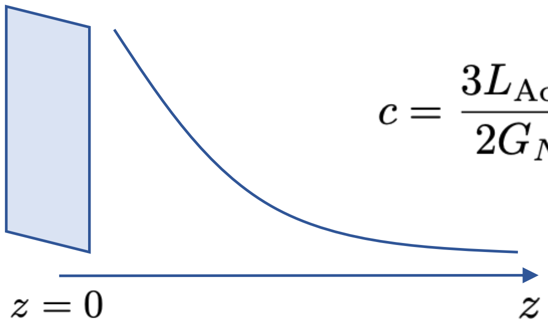
Einstein gravity on AdS_3 with matters at the classical limit	⟺	2d coset model with large central charge $\frac{\mathrm{SU}(2)_k \times \mathrm{SU}(2)_1}{\mathrm{SU}(2)_{k+1}}$

Central charge and the level of coset model

- A version of Gaberdiel-Gopakumar duality

Einstein gravity on AdS_3 with matters at the classical limit $\Longleftrightarrow$ 2d coset model with large central charge $\dfrac{SU(2)_k \times SU(2)_1}{SU(2)_{k+1}}$

- Comparison of central charge
  - Near the boundary of AdS_3 there appears Virasoro symmetry with central charge [Brown-Henneaux CMP '86]



$$c = \frac{3L_{\mathrm{AdS}}}{2G_N} \to \infty$$

  - The central charge of the coset is

$$c = 1 - \frac{6}{(k+2)(k+3)}$$

  - To have large central charge, we have to set

$$k \to -2 - \frac{6}{c} + \mathcal{O}(c^{-2})$$

Analytic continuation from AdS_3 to dS_3

[YH-Nishioka-Takayanagi-Taki PRL '22;JHEP '22]

- Formally we can move from AdS_3 to dS_3 by replacing $L_{\mathrm{AdS}} \to iL_{\mathrm{dS}}$
- Gaberdiel-Gopakumar duality becomes

Einstein gravity on dS_3 with matters at the classical limit	⟷	2d coset model with imaginary central charge $\frac{\mathrm{SU}(2)_k \times \mathrm{SU}(2)_1}{\mathrm{SU}(2)_{k+1}}$

- Comparison of central charge

$$c = 1 - \frac{6}{(k+2)(k+3)} = ic^{(g)}, \quad c^{(g)} = \frac{3L_{\mathrm{dS}}}{2G_N} \to \infty \quad \Longleftrightarrow \quad k \to -2 + i\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2})$$

At the leading order in $1/c^{(g)}$ only $SU(2)_k$ part dominates and the duality reduces to our proposal of dS_3/CFT_2 correspondence

Complex saddles of Chern-Simons gravity

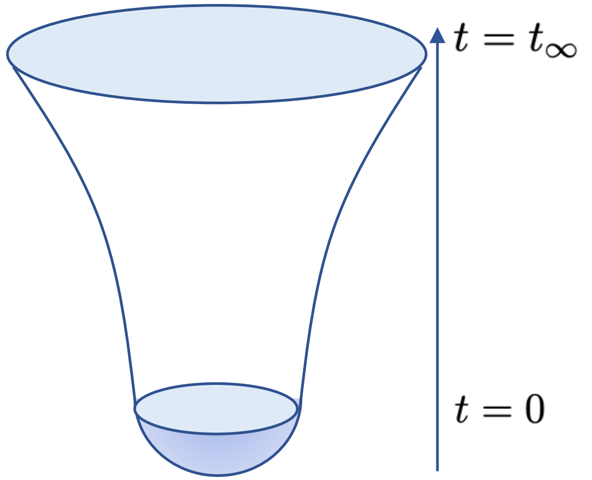
Allowable complex geometry

[Louko-Sorkin CQG ’97;Kontsevich-Segal QJM ’21;Witten’21]

- Allowable complex geometry
  - A complexified metric of S^{d+1}

$$ds^2 = L^2(\theta'(u)^2 du^2 + \cos^2\theta(u) d\Omega_d^2)$$

  - Universe can start from $\theta = (n + 1/2)\pi \ (n \in \mathbb{Z})$
  - A criteria of allowable geometry
    ➔ Only geometry with *n*=-1,0 are allowable
- Holographic method
  - We reproduce the same result via holography
  - We further consider more generic geometry



Complex saddles of Chern-Simons gravity

- We read off complex saddles of 3d Chern-Simons gravity via holography

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}], \ \ S_{\rm CS}[A] = -\frac{\kappa}{4\pi}\int \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

- The metric of dS_3 black hole is

$$ds^2 = L_{\rm dS}^2\left[(1-8G_N E_j - r^2)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right]$$

- Large gauge transformation generates different geometry labeled by *n*

$$ds^2 = L_{\rm dS}^2\left[(1-8G_N E_j - r^2)\left(2n+1\right)^2 d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right]$$

- Gravity partition function is a sum of contributions from each saddles

$$Z_{\rm dS} = \sum_n \exp S_{\rm GH}^{(n)}, \ \ S_{\rm GH}^{(n)} = \left(n+\frac{1}{2}\right)\frac{\pi L^2\sqrt{1-8G_N E}}{G_N}$$

Allowable geometry from dual CFT

[Chen-YH-Taki-Uetoko'23; in preparation]

- Dual CFT is given by Liouville theory with parameter b and the large central charge limit is realized by

$$c\,(\equiv ic^{(g)}) = 1 + 6(b + b^{-1})^2 \quad \Longrightarrow \quad b^{-2} = \frac{ic^{(g)}}{6} - \frac{13}{6} + \cdots$$

- dS_3 black hole is examined from 2-pt. function of heavy operator

$$\Psi_{\mathrm{dS}} \sim \langle V_\alpha(z_1) V_\alpha(z_2) \rangle$$

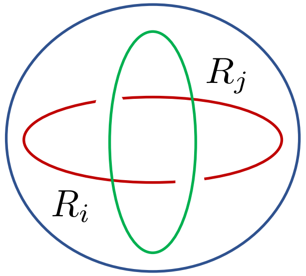
- Semi-classical expression of 2-pt. function can be read off from its exact result as [Harlow-Maltz-Witten'11]

$$|\langle V_\alpha(z_1) V_\alpha(z_2) \rangle| \sim e^{\frac{\pi}{6} c^{(g)} \sqrt{1 - 8 G_N E}} - e^{-\frac{\pi}{6} c^{(g)} \sqrt{1 - 8 G_N E}}$$

$\Longrightarrow$ We should pick up saddles of CS gravity with *n*=-1,0 and the result reproduces the allowable geometry of Witten

Geometry related to two linked Wilson loops

- GH entropy for the geometry corresponding to two lined Wilson loops on S^3 can be computed from *S*-matrix including non-perturbative corrections



$$\left| \cdots \right| = |\mathcal{S}_i{}^j| \simeq e^{\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_i}\sqrt{1-8G_N E_j}} - e^{-\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_i}\sqrt{1-8G_N E_j}}$$

- The result can be reproduced from Liouville four-point function

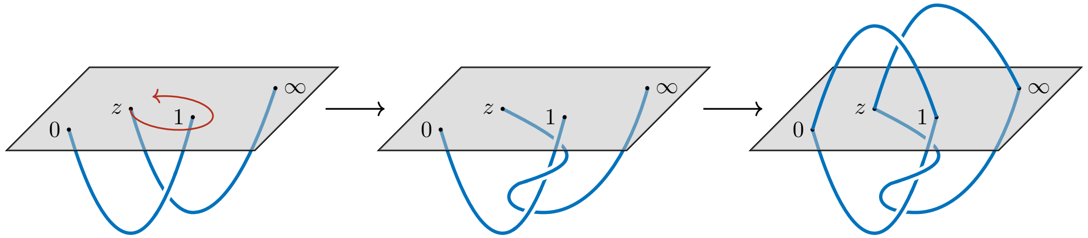
$$C_{ij}(z,\bar{z}) = \frac{\left\langle \mathcal{O}_i^\dagger(\infty)\mathcal{O}_j^\dagger(1)\mathcal{O}_i(z)\mathcal{O}_j(0)\right\rangle}{\left\langle \mathcal{O}_i^\dagger\mathcal{O}_i\right\rangle\left\langle \mathcal{O}_j^\dagger\mathcal{O}_j\right\rangle} = \sum_p \mathcal{F}_{jj}^{ii}(p|z)\bar{\mathcal{F}}_{jj}^{ii}(p|\bar{z})$$

Conformal blocks   Intermediate state $\mathcal{O}_p$

Linked Wilson loops from monodromy move

- Move of *z* around z = 1 yields monodromy matrix
  (cf. [Robert-Stanford PRL ’15;Caputa-Numasawa-Veliz-Osorio PTEP ’16; Fitzpatrick-Kaplan JHEP ’17])

$$\mathcal{F}^{ii}_{jj}(p|z) \to \sum_q \mathcal{M}_{pq}\mathcal{F}^{ii}_{jj}(q|z)$$



- Since *p*=0 dominates at large *c*, the combination of anti-holomorphic part leads to

$$C_{ij}(z,\bar{z}) \sim \mathcal{M}_{00}\mathcal{F}^{ii}_{jj}(0|z)\bar{\mathcal{F}}^{ii}_{jj}(0|\bar{z}),\ \mathcal{M}_{00} = \frac{S^*_{ij}S_{00}S_{00}}{S_{00}S_{0i}S_{0j}} \quad (S_{ij} \text{ is the modular } S\text{-matrix of Liouville theory})$$

- Up to normalization, the previous result is reproduced from the four-point function

$$\left|\left\langle \mathcal{O}^\dagger_i(\infty)\mathcal{O}^\dagger_j(1)\mathcal{O}_i(z)\mathcal{O}_j(0)\right\rangle\right| = |S_{ij}| \simeq e^{\frac{\pi c^{(g)}}{3}\sqrt{1-8G_NE_i}\sqrt{1-8G_NE_j}} + e^{-\frac{\pi c^{(g)}}{3}\sqrt{1-8G_NE_i}\sqrt{1-8G_NE_j}}$$

Conclusion

Summary & future problems

- Summary
  - dS/CFT correspondence is proposed between classical higher-spin dS_3 gravity and 2d SU(N) WZW model (Liouville theory) with large central charge
  - Evidence is provided by comparing partition functions and relating to higher-spin AdS_3 holography
  - Allowable complex geometries of Chern-Simons gravity are read off from dual CFT correlators
- Future problems
  - Examine Liouville/Toda multi-point functions [Chen-YH-Taki-Uetoko in preparation]
  - Read off quantum effects of dS_3 (higher-spin) gravity from dual CFT_2
  - Generalize the analysis to other dS/CFT correspondence (e.g., higher dimensions, stringy realizations, etc.)

Thank you for your attention