Psuedo Entanglement entropy in 2D CFTs

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Outline

- **Introduction of EE and Psuedo entanglement entropy**
- **Psuedo Renyi entropy in 2D CFT**

1. **Replic trick and setup**
2. **Psuedo Renyi entropy of locally excited states**

- **Reality conditions for Psuedo Renyi entropy**
- **Summary**

What is Pseudo entropy?

Def. Of EE in discrete systems

Divide a quantum system into two parts A and B.

$$H_{tot} = H_A \otimes H_B$$ **Factorization Property**

Example: Spin Chain

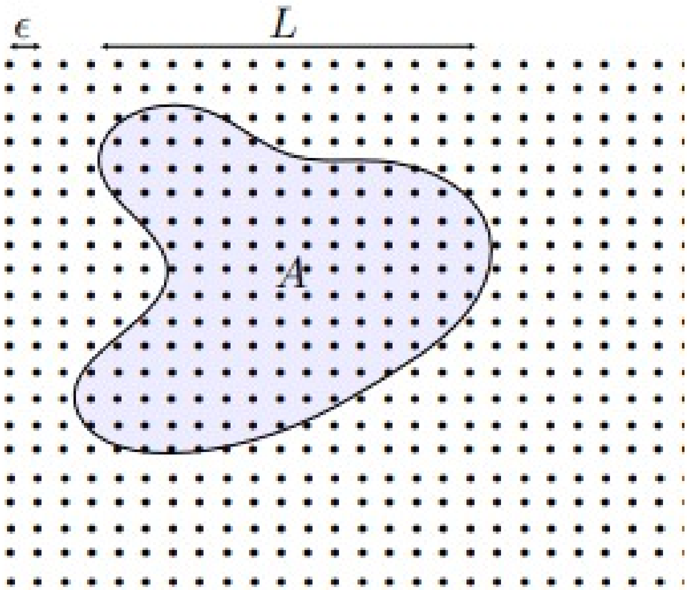
Reduced Density Matrix: $$\rho_A = \mathrm{Tr}_B \rho_{tot}$$

von-Neumann entropy : $$S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A$$

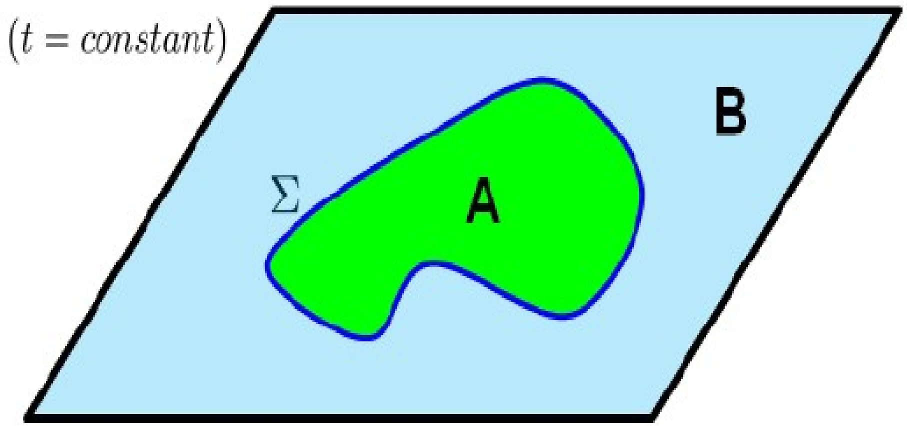
Fine grained Entropy

Definition of EE in QFT:

In QFTs, the EE is defined geometrically (called geometric entropy).



Continuum Limit $\epsilon \to 0$



$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

$$H_{tot} = H_A \otimes H_B$$

No factorization in Gauge theory and gravity!!

Definition of Transition matrix in QFT:

$$\mathcal{T}^{\psi|\varphi} \equiv \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}$$

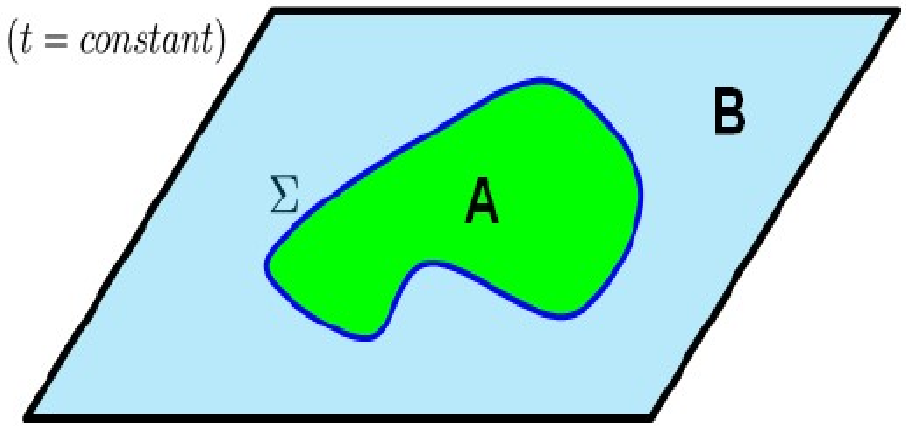
Properties:

$$\mathrm{Tr}\,\mathcal{T}^{\psi|\varphi} = 1$$

$$\left(\mathcal{T}^{\psi|\varphi}\right)^n = \mathcal{T}^{\psi|\varphi}, \quad \forall n \in \mathbb{N}^+$$

$$\mathrm{Tr}\left(\mathcal{T}^{\psi|\varphi}\right)^n = 1$$

$$\mathcal{T}^{\psi|\varphi} = \left(\mathcal{T}^{\varphi|\psi}\right)^\dagger$$



$$H_{tot} = H_A \otimes H_B$$

Reduced Transition matrix:

$$\mathcal{T}_A^{\psi|\varphi} = \mathrm{Tr}_B\left(\mathcal{T}^{\psi|\varphi}\right)$$

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② : Basic properties of Pseudo-(Rényi) Entropy

1) **If either $|\psi\rangle$ or $|\varphi\rangle$ is product state, then** $S^{(n)}(\mathcal{T}_A^{\psi|\varphi}) = 0$**.** $(\mathcal{T}_A^{\psi|\varphi} \equiv \mathrm{tr}_{\bar{A}}[\mathcal{T}^{\psi|\varphi}])$

$\bar{A}$: complement of A

2) $S^{(n)}(\mathcal{T}_A^{\psi|\varphi}) = S^{(n)}(\mathcal{T}_{\bar{A}}^{\psi|\varphi})$

(1) and 2): Like EE)

3) $S^{(n)}(\mathcal{T}_A^{\psi|\varphi}) = \left[S^{(n)}(\mathcal{T}_A^{\varphi|\psi})\right]^*$

4) $\mathcal{T}_A^{\psi|\varphi}$ **is non-Hermitian in general, pseudo-entropy can be complex-valued.**

Subadditivity and Strong Subadditivity are violated in general!

Introduction: Pseudo-(Rényi) entropy

$$\mathcal{T}^{\psi_1|\psi_2} = \frac{|\psi_1\rangle\langle\psi_2|}{\langle\psi_2|\psi_1\rangle}$$

Transition matrix ($\langle\psi_2|\psi_1\rangle \neq 0$)

Partial trace

$$\mathcal{T}_A^{\psi_1|\psi_2} = \mathrm{Tr}_{A^c}[\mathcal{T}^{\psi_1|\psi_2}]$$

Reduced transition matrix

PE: $$S_A = -\mathrm{Tr}[\mathcal{T}_A^{\psi_1|\psi_2} \log \mathcal{T}_A^{\psi_1|\psi_2}]$$

We can also define the corresponding "pseudo-Rényi entropy (PRE)" with respect to $\mathcal{T}_A^{\psi_1|\psi_2}$

T. Takayanagi et al '20

PRE: $$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}[(\mathcal{T}_A^{\psi_1|\psi_2})^n] \quad n \in \mathbb{R}^+ \setminus \{1\}$$

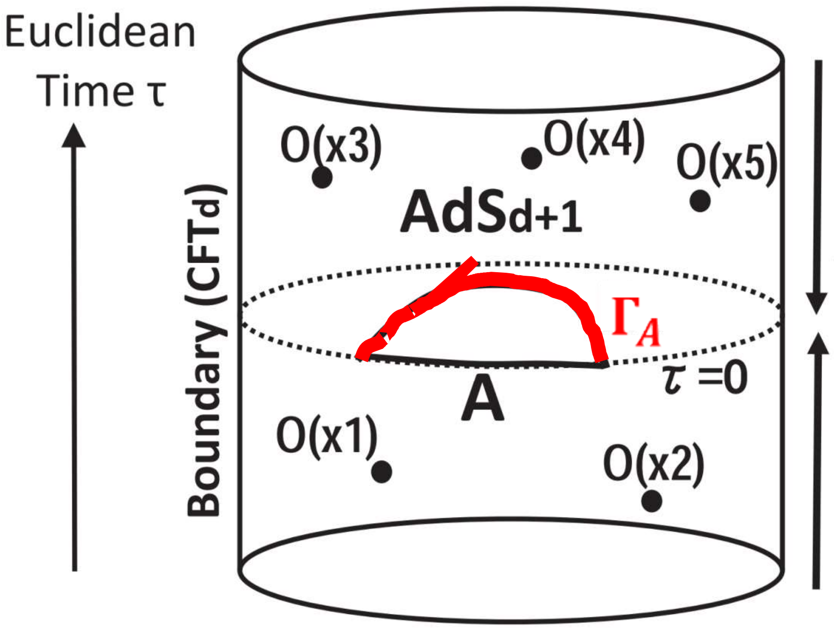
$$\lim_{n\to 1} S_A^{(n)} = S_A \quad ✓$$

PE and PRE are normally complex!

③ : Interpretation from Holography

[Nakata, Takayanagi, Taki, Tamaoka, Wei'2020]

In AdS/CFT correspondence, pseudo-entropy is dual to **area of minimal surfaces** in time-dependent Euclidean asymptotically AdS (aAdS) spaces



$$|\varphi\rangle = O(x_5)O(x_4)O(x_3)|0\rangle$$

$$|\psi\rangle = O(x_2)O(x_1)|0\rangle$$

$$Z_{\text{gravity}} = \langle\varphi|\psi\rangle$$

(GKP-W relation)

aAdS side:

$$S_A = \min_{\substack{\Gamma_A \\ \partial\Gamma_A = \partial A}} \left[\frac{\text{Area}[\Gamma_A]}{4G}\right]$$

=

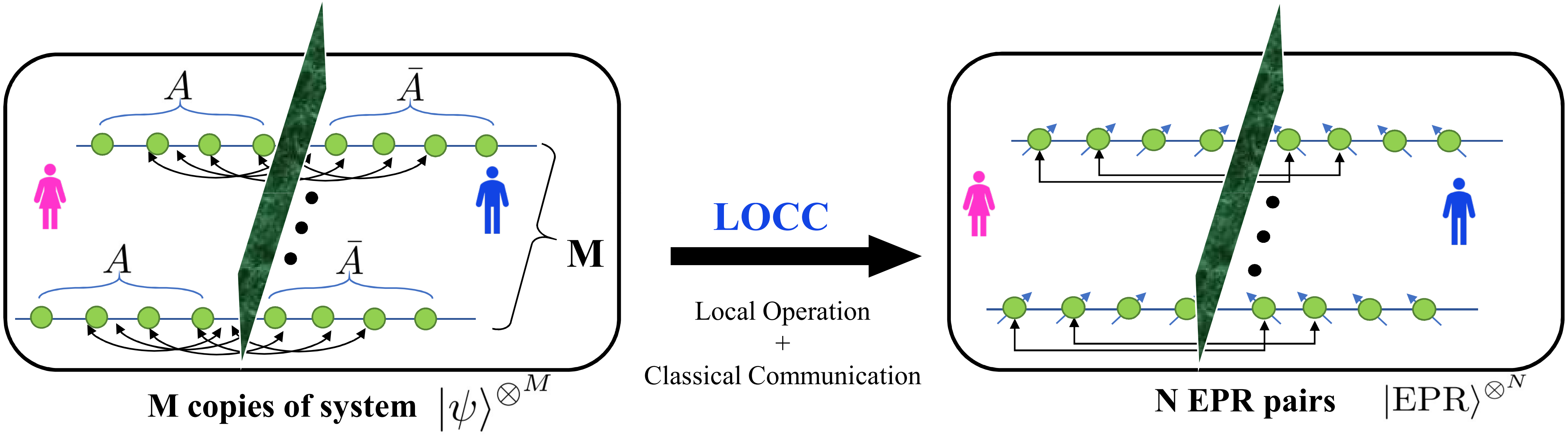
CFT side:

$$S_A = -\text{tr}[\mathcal{T}_A^{\psi|\varphi} \log \mathcal{T}_A^{\psi|\varphi}]$$

③ : Interpretation from Quantum Entanglement

[Bennett, Bernstein, Popescu, Schumacher'1995]

Entanglement entropy = **# of distillable EPR pairs** under **LOCC**

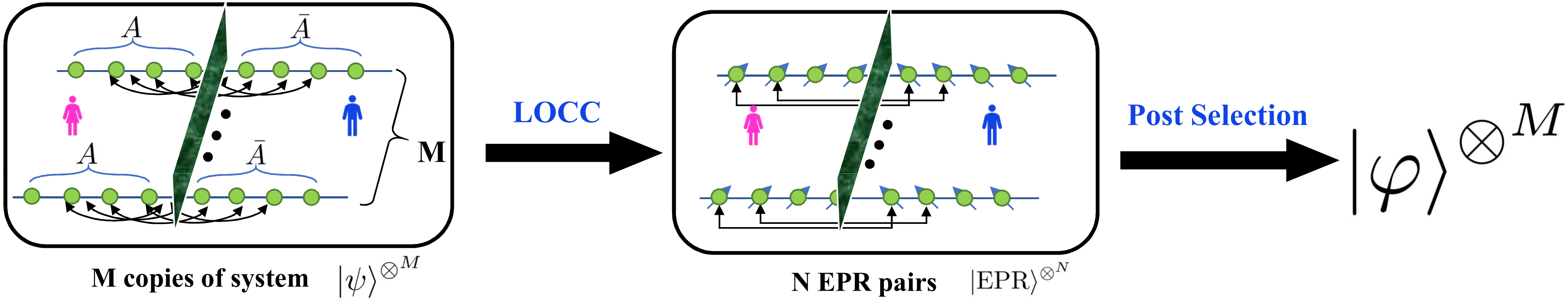


EE: $$S(\rho_A^{\psi}) = \lim_{M\to\infty} \frac{N}{M}$$ $(\rho_A^{\psi} = \mathrm{tr}_{\bar{A}}|\psi\rangle\langle\psi|)$

③ : Interpretation from Quantum Entanglement

[Nakata, Takayanagi, Taki, Tamaoka, Wei'2020]

Pseudo-entropy = ⟨#⟩ **of distillable EPR pairs** under **LOCC+Post Selection**

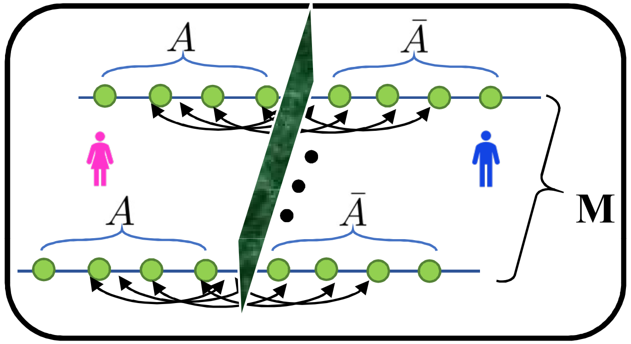


PE: $$S(\mathcal{T}_A^{\psi|\varphi}) = \lim_{M\to\infty} \frac{\langle N\rangle}{M}$$

③ : Interpretation from Quantum Entanglement

[Nakata, Takayanagi, Taki, Tamaoka, Wei'2020]

Pseudo-entropy = **# of distillable** [



M copies of system $|\psi\rangle^{\otimes M}$

LOCC

$\langle \boldsymbol{N} \rangle$

=

The **average** of **N** with respect to **various** **LOCCs**

$\otimes M$

PE: $$S(\mathcal{T}_A^{\psi|\varphi}) = \lim_{M\to\infty} \frac{\langle N\rangle}{M}$$

Pseudo entropy of locally excited states in 2D CFTs

PRE of 2D CFTs in real time (Our focus)

What happens if we think about the pseudo-(Rényi) entropy in real-time?

We consider

$$\mathcal{T}_A^{\psi_1|\psi_2}(t) = \frac{\mathrm{e}^{-iHt}|\psi_1\rangle\langle\psi_2|\mathrm{e}^{iHt}}{\langle\psi_2|\psi_1\rangle}$$

$$\Downarrow$$

$$S_A^{(n)}(t) = \frac{1}{1-n}\log \mathrm{Tr}[(\mathcal{T}_A^{\psi_1|\psi_2}(t))^n]$$

PRE of 2D CFTs in real time

One simple scheme in 2d CFTs is to consider two locally excited states:

Regulator

$$|\psi_i\rangle = e^{-\epsilon H}\mathcal{O}_i(t=0,x_i)|\Omega\rangle$$

Primay, descendent …

Vacuum

$$\mathcal{T}_A^{\psi_1|\psi_2}(t) = \frac{\mathrm{e}^{-iHt}|\psi_1\rangle\langle\psi_2|\mathrm{e}^{iHt}}{\langle\psi_2|\psi_1\rangle}$$

$$S_A^{(n)}(t) = \frac{1}{1-n}\log \mathrm{Tr}[(\mathcal{T}_A^{\psi_1|\psi_2}(t))^n]$$

PRE for locally excited state: Replica trick

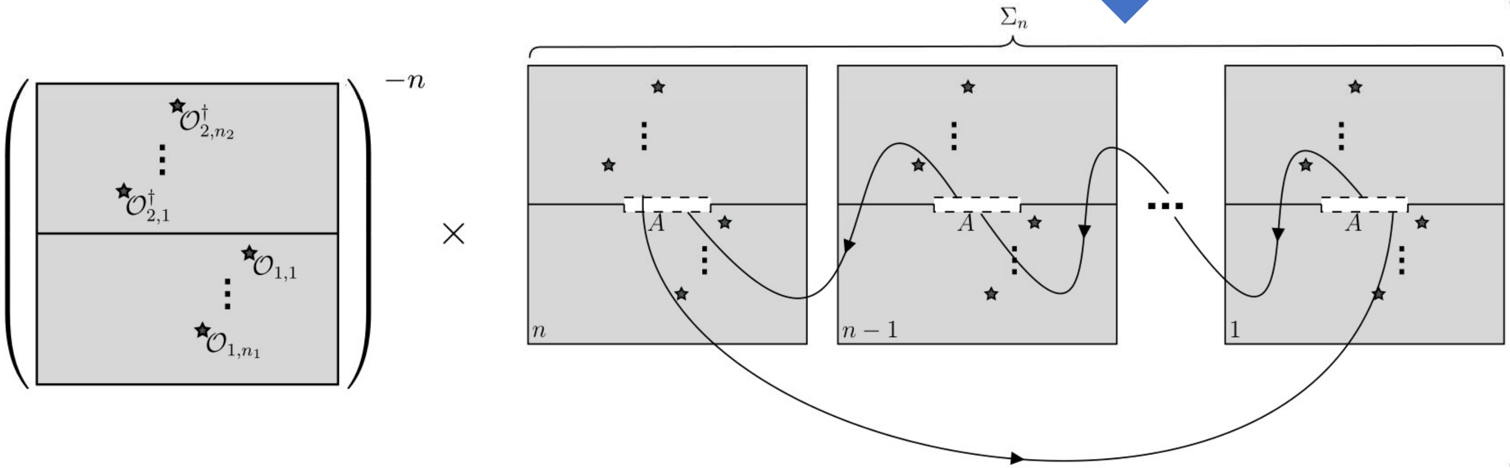
$$|\psi_j\rangle = \frac{1}{\sqrt{\mathcal{N}_j}} \mathcal{O}_{j,1}(-\tau_{j,1}, x_{j,1}) \mathcal{O}_{j,2}(-\tau_{j,2}, x_{j,2}) \ldots \mathcal{O}_{j,n_j}(-\tau_{j,n_j}, x_{j,n_j}) |\Omega\rangle,$$

$$(\tau_{j,i+1} \geq \tau_{j,i} > 0, \ i = 1, 2, \ldots, n_j - 1; \ j = 1, 2),$$

$$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}[(\mathcal{T}_A^{\psi_1|\psi_2})^n]$$

Euclidean Path Integral

$$S_A^{(n)} = \frac{1}{1-n} \log \left[\left(\ldots \right)^{-n} \times \ldots \right]$$



$$= S_{A;vac}^{(n)} + \frac{1}{1-n} \left(\log \left\langle (\mathcal{O}^{\dagger(n)}_{2,n_2} \ldots \mathcal{O}^{(n)}_{1,n_1}) \ldots (\mathcal{O}^{\dagger(1)}_{2,n_2} \ldots \mathcal{O}^{(1)}_{1,n_1}) \right\rangle_{\Sigma_n} - n \log \left\langle \mathcal{O}^\dagger_{2,n_2} \ldots \mathcal{O}_{1,n_1} \right\rangle_{\Sigma_1} \right).$$

$(n_1 + n_2) * n$-point function on Σ_n

$(n_1 + n_2)$-point function on Σ_1

PRE for locally excited state: Single primary

For $n=2$, $n_1=n_2=1$, $\Delta S_A^{(2)}$ is reduced to 4-point functions on Σ_2.

We further assume $\mathcal{O}_1=\mathcal{O}_2=\mathcal{O}$ to simplify the results

$$\Delta S_A^{(2)} = -\log \frac{\left\langle \mathcal{O}^{\dagger(2)}(\tau_2,x_2)\mathcal{O}^{(2)}(-\tau_1,x_1)\mathcal{O}^{\dagger(1)}(\tau_2,x_2)\mathcal{O}^{(1)}(-\tau_1,x_1)\right\rangle_{\Sigma_2}}{\left\langle \mathcal{O}^{\dagger}(\tau_2,x_2)\mathcal{O}(-\tau_1,x_1)\right\rangle_{\Sigma_1}^2}$$

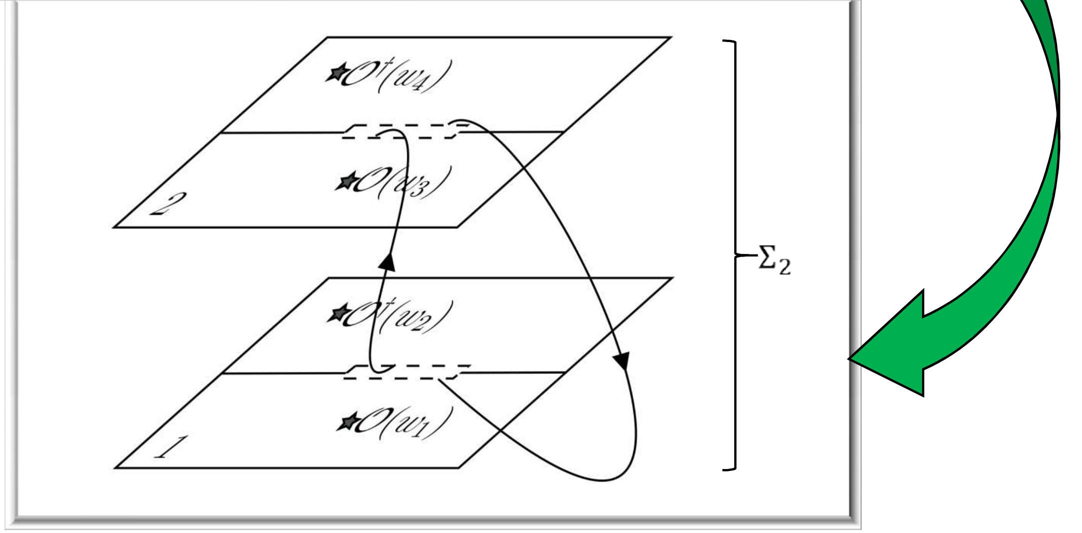
Note1: Conformal map between Σ_2 and Σ_1

$$z = \left(\frac{w}{w-L}\right)^{1/n}, \quad (A=[0,L]),$$

$$z = w^{1/n}, \quad (A=[0,+\infty))$$

Note2: Analytic continuation of t

$$\tau_1 = \epsilon + it, \quad \tau_2 = \epsilon - it$$



PRE for locally excited state: Single primary

$$\Delta S_A^{(2)} = -\log \frac{\langle \mathcal{O}^{\dagger(2)}(\tau_2, x_2)\mathcal{O}^{(2)}(-\tau_1, x_1)\mathcal{O}^{\dagger(1)}(\tau_2, x_2)\mathcal{O}^{(1)}(-\tau_1, x_1)\rangle_{\Sigma_2}}{\langle \mathcal{O}^{\dagger}(\tau_2, x_2)\mathcal{O}(-\tau_1, x_1)\rangle_{\Sigma_1}^2}$$

$$z = \left(\frac{w}{w-L}\right)^{1/n}, \quad (A = [0, L]),$$

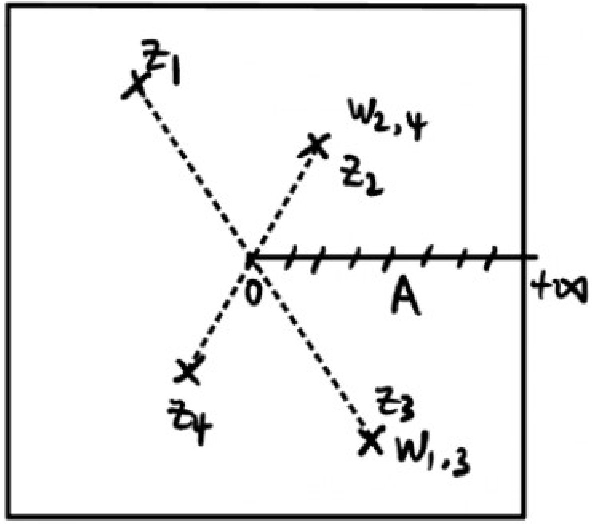
$$z = w^{1/n}, \quad (A = [0, +\infty))$$

$$(w_3, \bar{w}_3)_{\text{sheet 2}} = (w_1, \bar{w}_1)_{\text{sheet 1}} = (x_1 - i\tau_1, x_1 + i\tau_1)$$

$$(w_4, \bar{w}_4)_{\text{sheet 2}} = (w_2, \bar{w}_2)_{\text{sheet 1}} = (x_2 + i\tau_2, x_2 - i\tau_2)$$

$$\langle \phi_1(\vec{x}_1)\phi_2(\vec{x}_2)\phi_3(\vec{x}_3)\phi_4(\vec{x}_4)\rangle = f(\eta, \bar{\eta}) \prod_{i<j}^{4} z_{ij}^{\frac{h}{3} - h_i - h_j} \bar{z}_{ij}^{\frac{\bar{h}}{3} - \bar{h}_i - \bar{h}_j}$$

$$(\eta, \bar{\eta}) = \left(\frac{z_{12}z_{34}}{z_{13}z_{24}}, \frac{\bar{z}_{12}\bar{z}_{34}}{\bar{z}_{13}\bar{z}_{24}}\right)$$



PRE for locally excited state: Single primary

$$\Delta S_A^{(2)} = \log \frac{\left\langle \mathcal{O}^{\dagger(2)}(\tau_2, x_2)\mathcal{O}^{(2)}(-\tau_1, x_1)\mathcal{O}^{\dagger(1)}(\tau_2, x_2)\mathcal{O}^{(1)}(-\tau_1, x_1)\right\rangle_{\Sigma_2}}{\left\langle \mathcal{O}^{\dagger}(\tau_2, x_2)\mathcal{O}(-\tau_1, x_1)\right\rangle_{\Sigma_1}^2}$$

$$z = \left(\frac{w}{w-L}\right)^{\frac{1}{2}}, \quad A = [0, L]$$

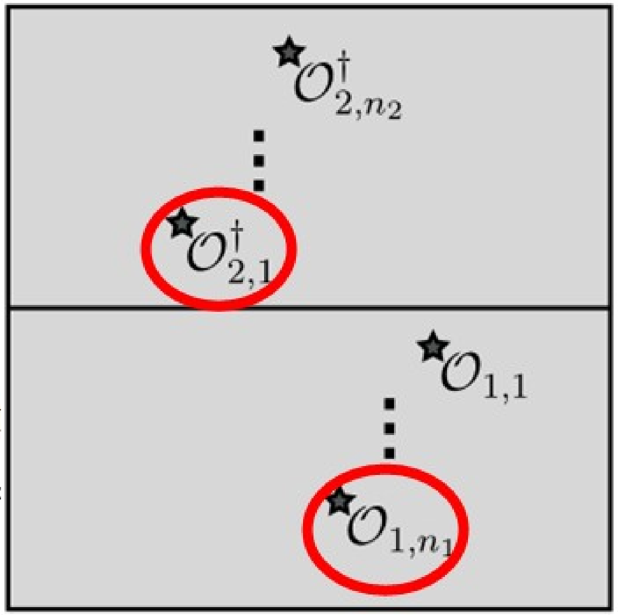
$$z = w^{\frac{1}{2}}, \quad A = [0, +\infty)$$

$$\langle \mathcal{O}^{\dagger}(z_2, \bar{z}_2)\mathcal{O}(z_1, \bar{z}_1)\rangle_{\Sigma_1} = \frac{c_{12}}{|z_{12}|^{4\Delta_{\mathcal{O}}}}$$

$$\langle \mathcal{O}^{\dagger(2)}(\tau_2, x_2)\mathcal{O}^{(2)}(-\tau_1, x_1)\mathcal{O}^{\dagger(1)}(\tau_2, x_2)\mathcal{O}^{(1)}(-\tau_1, x_1)\rangle_{\Sigma_2} = |16 z_1^2 z_2^2|^{-4\Delta_{\mathcal{O}}} G(\eta, \bar{\eta})$$

$$\Delta S_A^{(2)} = \log \frac{c_{12}^2}{|\eta(1-\eta)|^{4\Delta_{\mathcal{O}}} \cdot G(\eta, \bar{\eta})}$$

PRE for locally excited state: Single primary



$$A = [0, \infty).$$

Table 1: Early time and late time behaviors of $(\eta, \bar{\eta})$ for the subsyster

$(\eta, \bar{\eta})$	$x_1 x_2 > 0$	$x_1 x_2 < 0$	
Late time $(t \to \infty)$	$(1, 0)$	$(1, 0)$	
Early time $(t \to 0)$	$(\frac{1}{2} + a, \frac{1}{2} + a)$ $a = \frac{x_1 + x_2}{4\sqrt{x_1 x_2}}$	$x_1 > 0 > x_2$	$x_2 > 0 > x_1$
		$(\frac{1}{2} + a, \frac{1}{2} - a)$	$(\frac{1}{2} - a, \frac{1}{2} + a)$

PRE for locally excited state: Single primary

Late time limit ($A = [0, \infty)$):

Quantum dimension of $\mathcal{O}$

Rational CFTs: $$\Delta S_A^{(2)} \simeq \begin{cases} 0, & t \to 0 \ \&\& \ x_1 \sim x_2, \\ \log d_{\mathcal{O}}, & t \to \infty. \end{cases}$$

S. He et al' 14

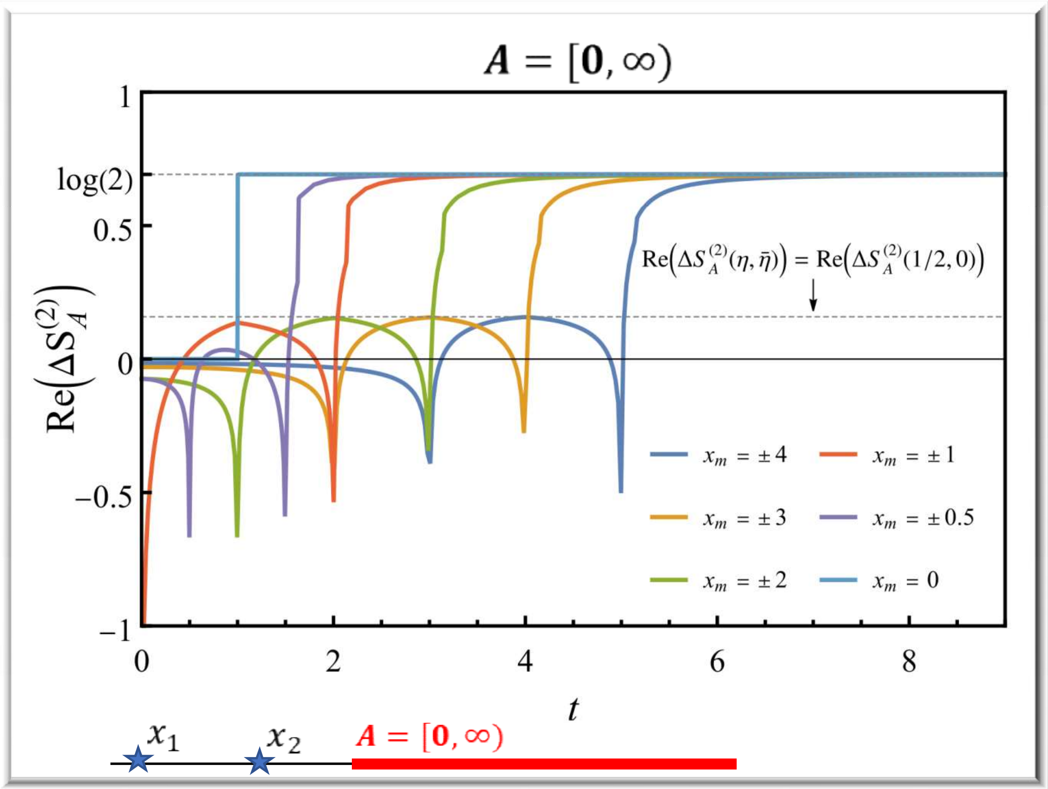
Large-c CFTs: $(c \to \infty)$ $$\mathrm{Re}\left[\Delta S_A^{(2)}\right] = 4\Delta_{\mathcal{O}} \log \frac{4t}{\sqrt{(x_1 - x_2)^2 + 4\epsilon^2}}$$

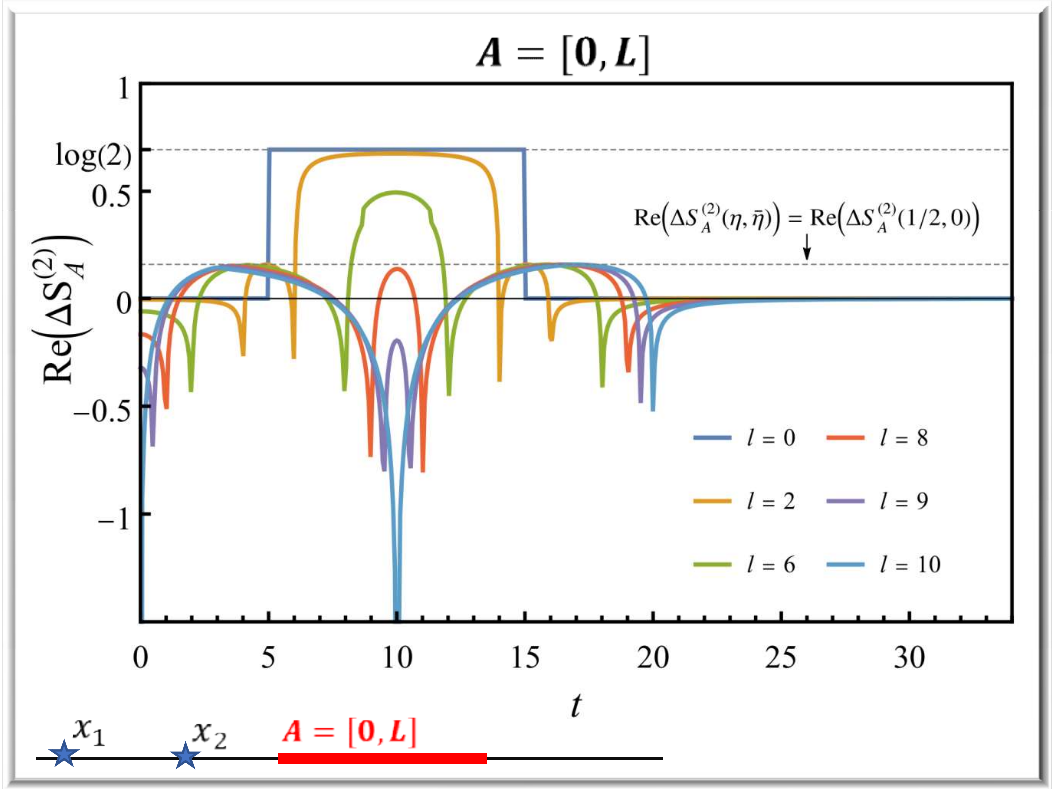
P. Caputa et al' 15

PRE for locally excited state: Single primary

Full-time evolution: $\mathcal{O}=(e^{\frac{i}{2}\phi}+e^{-\frac{i}{2}\phi})$-excitation in free scalar

$d_{\mathcal{O}}=2$

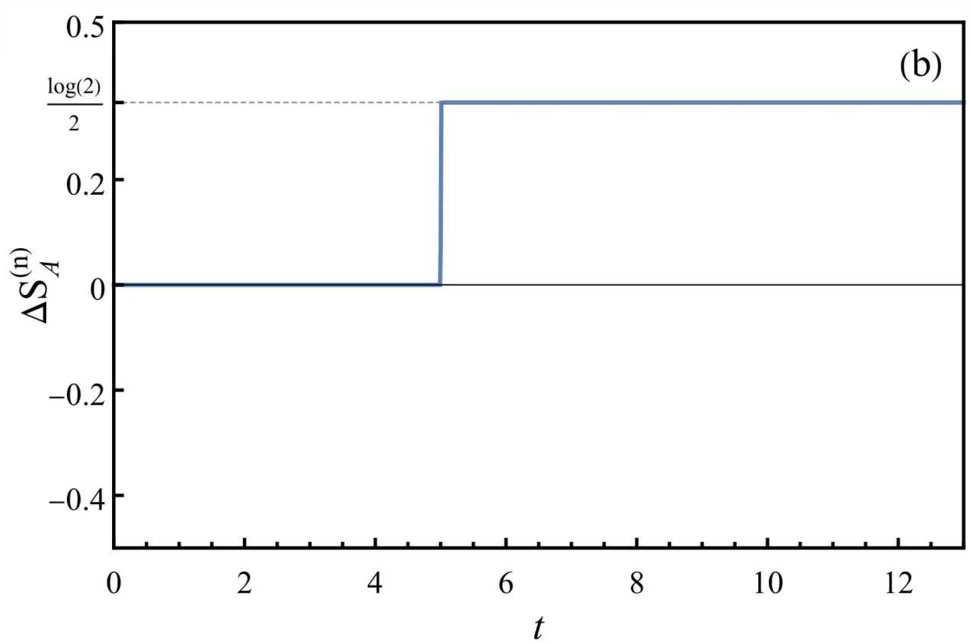




PRE for locally excited state: Single primary

When $A = [0, \infty)$, the late time limit of $\log d_{\mathcal{O}}$ is true for any order of $\Delta S_A^{(n)}$.

$$\lim_{t\to\infty} \Delta S_A^{(n)} = \log d_{\mathcal{O}}$$



Example:

σ(spin)-excitation in critical Ising model

$$\Delta S^{(n)}_{A=[0,\infty)}(x, -x, t) = \begin{cases} 0, & 0 \le t < |x|, \\ \log d, & t > |x|. \end{cases}$$

Reality condition of Pseudo entropy

④ : Brief Summary

Non-Negative Pseudo-Entropy Requirement

For HPE, **the holographic calculation** requires $\mathcal{T}_A^{\psi|\varphi}$ to generate **non-negative** $S(\mathcal{T}_A^{\psi|\varphi})$.

The proof of **QI interpretation for PE** **is valid** when $\mathcal{T}_A^{\psi|\varphi}$ **is semi-positive definite.**

However

It's usually **Non-Hermitian**

$$S_A = -\mathrm{tr}[\mathcal{T}_A^{\psi|\varphi} \log \mathcal{T}_A^{\psi|\varphi}]$$

Complex-valued in general!

⑤ : Our focus and motivation

Our motivation

To find the **sufficient and necessary condition**
for
the **transition matrix** $\mathcal{T}^{\psi|\varphi}$ such that $S^{(n)}(\mathcal{T}_A^{\psi|\varphi}) \geq 0$?

⑥ : Resort to Pseudo-Hermiticity due to matrix algebra

What is Pseudo-Hermiticity

An operator M is said to be **η-pseudo-Hermitian** if there **exists a Hermitian invertible operator** η such that

$$M^\dagger = \eta M \eta^{-1}.$$

[Mostafazadeh'2001]

★ If $\eta = \mathbb{I}$, the pseudo-Hermitian condition reduces to the Hermitian condition.

Pseudo-Hemiticity: A generalization of Hermiticity.

⑦ : Basic Properties of Pseudo-Hermiticity

Property 1:

Any operator $\mathcal{O}$ can be decomposed into the sum of **two** η**-pseudo-Hermitian operators**

Any invertible Hermitian operator

$$\mathcal{O} = \mathcal{O}_1 + i\mathcal{O}_2,$$

Proof:

$$\mathcal{O} = \frac{\mathcal{O} + \eta^{-1}\mathcal{O}^\dagger\eta}{2} + i\frac{\mathcal{O} - \eta^{-1}\mathcal{O}^\dagger\eta}{2i},$$

Property 2:

Hint: Construct η using biorthonormal bases

Suppose that $\mathcal{O}$ **is diagonalizable**, then $\mathcal{O}$ **is** η**-pseudo-Hermitian** ***iff*** **the eigenvalues of** $\mathcal{O}$ **come in real numbers** or **complex conjugate pairs**

[Mostafazadeh'2001]

① : Summary of Our Results

*We still don't know how to construct this set

Subset

$\mathcal{T}^{\psi|\varphi}$ are η-pseudo-Hermitian

$(\eta = \eta_A \otimes \eta_{\bar{A}})$

+

Both η_A and $\eta_{\bar{A}}$ are positive or negative

$$\left\{ \mathcal{T}^{\psi|\varphi} \,\middle|\, S^{(n)}(\mathcal{T}_A^{\psi|\varphi}) \geq 0 \right\}$$

We find the **blue subset** which gives non-negative Pseudo-Rényi entropies

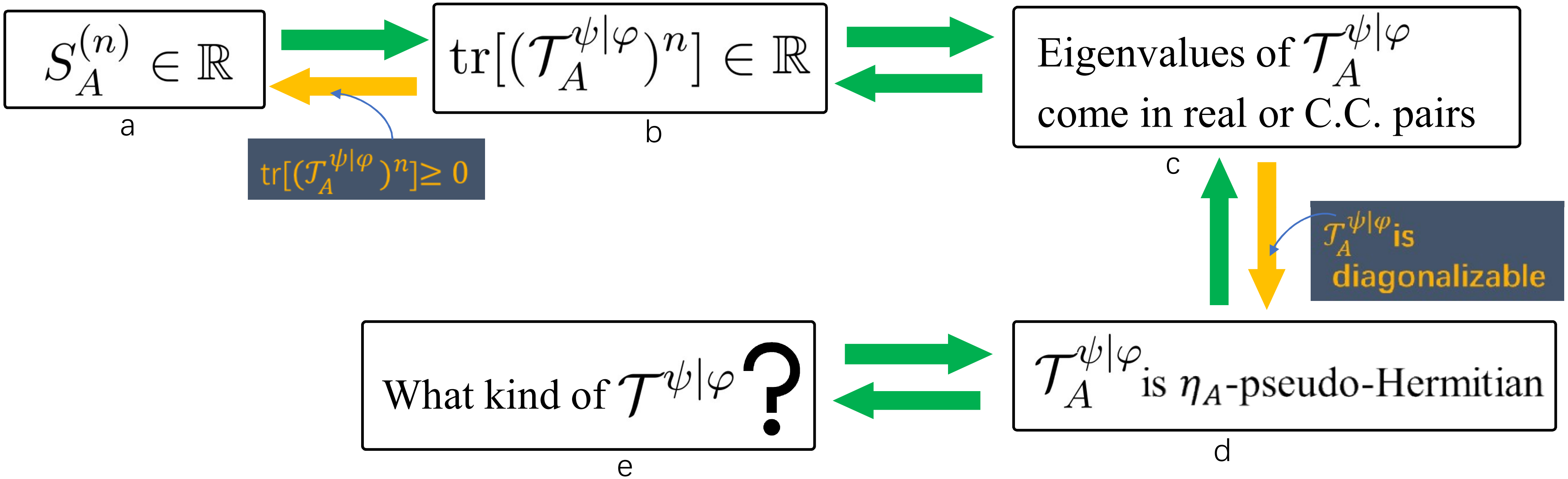
★**All density matrices** belong to the blue subset!

② : Mind Mapping of the Construction

$$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}[(\mathcal{T}_A^{\psi_1|\psi_2})^n]$$

$n \in \mathbb{R}^+ \setminus \{1\}$

Chart of equivalence relation



n-th Pseudo-Rényi entropy: $S_A^{(n)} = \frac{1}{1-n} \log \mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n]$

③ : Find $\mathcal{T}^{\psi|\varphi}$ that generates **pseudo-Hermitian** $\mathcal{T}_A^{\psi|\varphi}$

e | What kind of $\mathcal{T}^{\psi|\varphi}$? | ⇄ | $\mathcal{T}_A^{\psi|\varphi}$ is η_A-pseudo-Hermitian | d

↓

The answer:

Theorem 1:

T can be written as $T = T_1 + iT_2$, where T_1 and T_2 are **both $\boldsymbol{\eta}$-pseudo-Hermitian**

with $\eta = \eta_A \otimes \eta_{\bar{A}}$. Further, T_2 satisfies $tr_{\bar{A}(A)}[T_2] = 0$.

[He, Guo, Zhang'2022]

(Simple proof in the next slide)

③ : Find $\mathcal{T}^{\psi|\varphi}$ that generates **pseudo-Hermitian** $\mathcal{T}_A^{\psi|\varphi}$

Theorem 1

$X_{A(\bar{A})}$ is $\eta_{A(\bar{A})}$-**pseudo-Hermitian**, *iff* X can be written as $X = X_1 + iX_2$, where X_1 and X_2 are **both $\boldsymbol{\eta}$-pseudo-Hermitian** with $\eta = \eta_A \otimes \eta_{\bar{A}}$. Further, X_2 satisfies $tr_{\bar{A}(A)}[X_2] = 0$.

Proof:

$$tr_{\bar{A}}[X_2] = tr_{\bar{A}}\left[\frac{X - \eta^{-1}X^{\dagger}\eta}{2}\right]$$

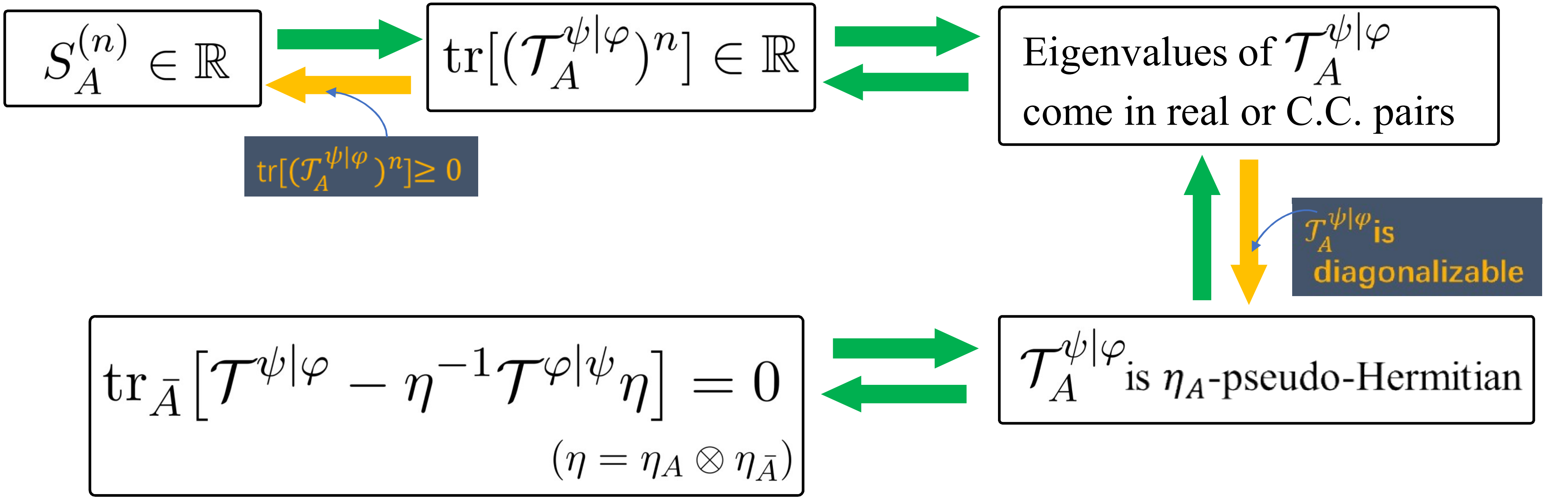
$$= \frac{1}{2}\left(X_A - \eta_A^{-1} tr_{\bar{A}}[\eta_{\bar{A}}^{-1}X^{\dagger}\eta_{\bar{A}}]\eta_A\right)$$

$$= \frac{1}{2}\left(X_A - \eta_A^{-1}X_A^{\dagger}\eta_A\right) = 0$$

⬅ **pseudo-Hermitian**

④ : Constructions

Chart of equivalence relation



n-th Pseudo-Rényi entropy: $$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n]$$

④ : Barriers

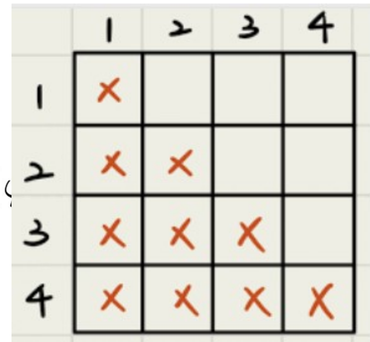
Ch… n

$S_A^{(n)} \in \mathbb{R}$

tr$[(\mathcal{T}_A$…

$\mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n] \geq 0$

$\mathcal{T}^{\psi|\varphi}$…



Eigenvalues of $\mathcal{T}_A^{\psi|\varphi}$ come in real or C.C. pairs

$\mathcal{T}_A^{\psi|\varphi}$ is diagonalizable

$$\mathrm{tr}_{\bar{A}}\left[\mathcal{T}^{\psi|\varphi} - \eta^{-1}\mathcal{T}^{\varphi|\psi}\eta\right] = 0$$

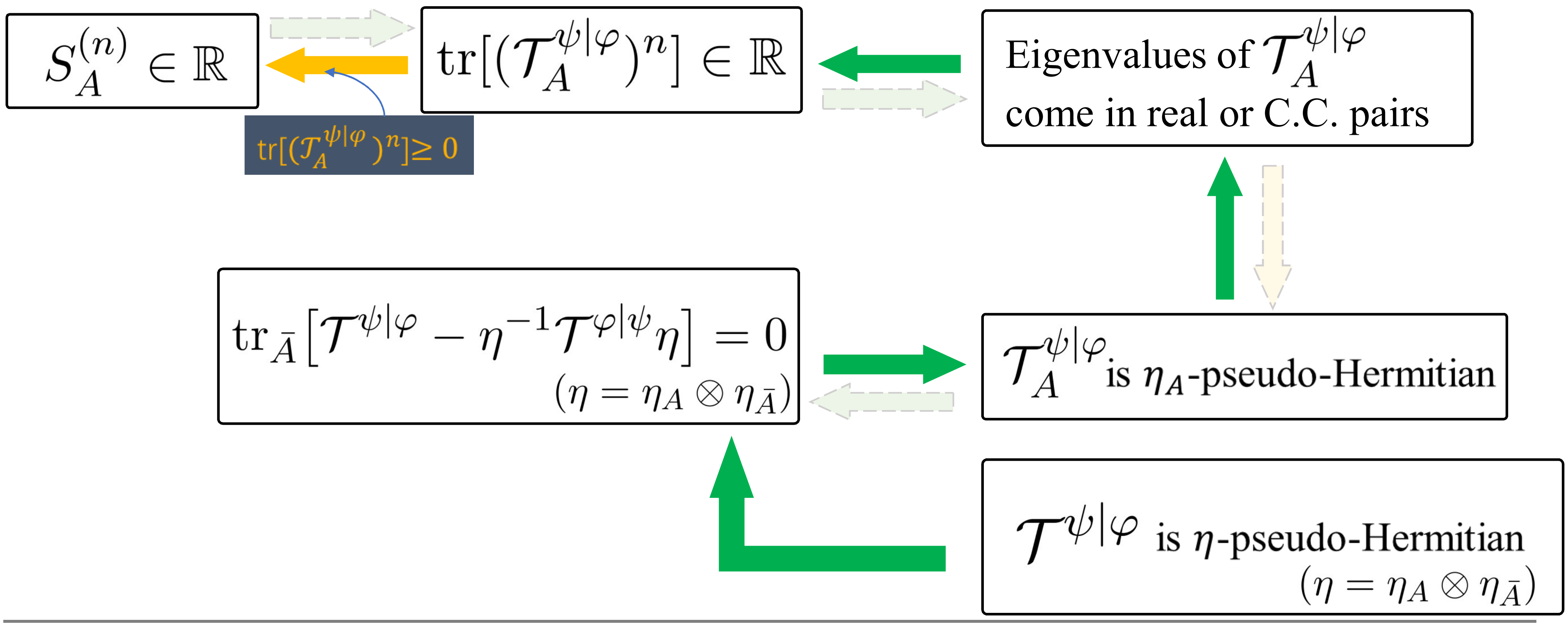
$(\eta = \eta_A \otimes \eta_{\bar{A}})$

$\mathcal{T}_A^{\psi|\varphi}$ is η_A-pseudo-Hermitian

n-th Pseudo-Rényi entropy: $S_A^{(n)} = \dfrac{1}{1-n} \log \mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n]$

⑤ : The Sufficient Condition

Chart of equivalence relation



⑤ : Find the Sufficient Condition

Chart of equivalen

$S_A^{(n)} \in \mathbb{R}$

$\mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n] \in \mathbb{R}$

$\mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n] \geq 0$

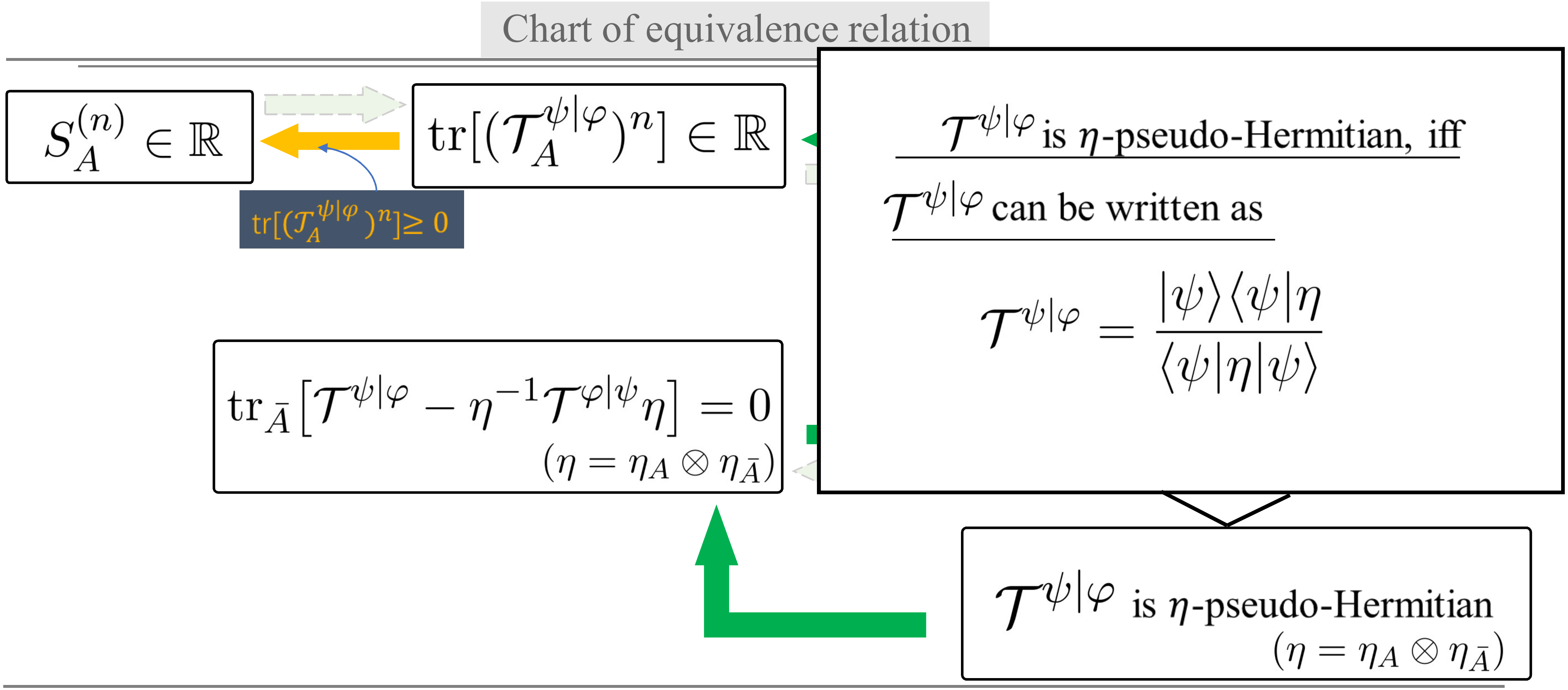
This condition can be satisfied by imposing some additional constraints on η!

$\mathrm{tr}_{\bar{A}}\left[\mathcal{T}^{\psi|\varphi} - \eta^{-1}\mathcal{T}^{\varphi|\psi}\eta\right] = 0$ $(\eta = \eta_A \otimes \eta_{\bar{A}})$

$\mathcal{T}_A^{\psi|\varphi}$ is η_A-pseudo-Hermitian

$\mathcal{T}^{\psi|\varphi}$ is η-pseudo-Hermitian $(\eta = \eta_A \otimes \eta_{\bar{A}})$

⑤ : Find the Sufficient Condition



⑤ : The Sufficient Condition

Theorem 2:

Suppose $\mathcal{T}^{\psi|\varphi}$ is η-pseudo-Hermitian with $\eta = \eta_A \otimes \eta_{\bar{A}}$:

If η_A is positive or negative and $\eta_{\bar{A}}$ is positive or negative too, then the eigenvalues of $\mathcal{T}^{\psi|\varphi}_{A(\bar{A})}$ are non-negative.

[He, Guo,Zhang'2022]

Hint of the Proof:

$$\eta_A^{1/2}\mathcal{T}_A^{\psi|\varphi}\eta_A^{-1/2} = \boxed{\eta_A^{1/2}\tilde{\mathcal{T}}_A^{\psi|\varphi}\eta_A^{1/2}}$$

Always Positive semi-definite!

$$\left(\tilde{\mathcal{T}}_A^{\psi|\varphi} = \frac{\mathrm{tr}_{\bar{A}}\left(\eta_{\bar{A}}^{1/2}|\psi\rangle\langle\psi|\eta_{\bar{A}}^{1/2}\right)}{\langle\psi|\eta|\psi\rangle}\right)$$

⑤ : The Sufficient Conditions

$\mathcal{T}^{\psi|\varphi}$ are η-pseudo-Hermitian

+ $(\eta = \eta_A \otimes \eta_{\bar{A}})$

Both η_A and $\eta_{\bar{A}}$ are positive or negative

☺

↓

Eigenvalues of $\mathcal{T}_A^{\psi|\varphi}$ are non-negative $(0 \leq \lambda_i \leq 1,\ \sum \lambda_i = 1)$

→

$$S^{(n)}(\mathcal{T}_A^{\psi|\varphi}) \geq 0$$

Part 5: Summary

Summary

- **Obtain Psuedo Renyi entropy of pair of locally excited states in 2D CFT.**
- **Late time of PRE is log quantum dimension (Universal)**
- **Construct the sufficient condition for PRE**
- **PRE for more generic locally excited states, Please refer to our work.**
- **How to extend the reality condition for Type III &Type II algebra, PRE in SYK, etc, …**

Thanks for your attention