Lightlike form factors and OPE

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Based on the work with Yuanhong Guo, Lei Wang
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Outline:

I. Introduction

II. Bootstrap ~ 2-loop $F_4^{(2)}$

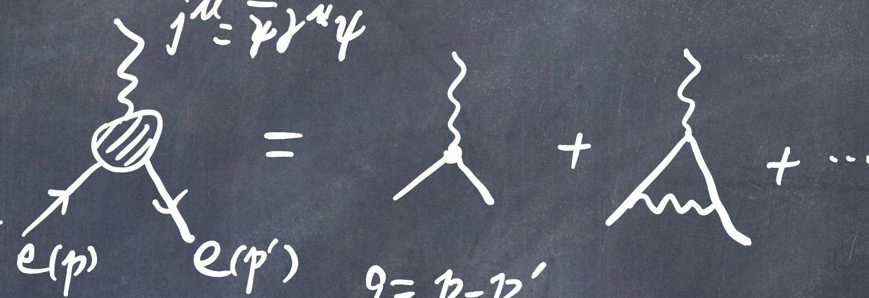
III. FFOPE ~ Non-perturbative

IV. Summary & Outlook.

I. Introduction and some history.

What are form factors?

EM vectex :

$\Gamma^\mu =$ 

$q = p - p'$

$$= \gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2)$$

Form factor.

$$g-2 = 2F_2(q^2=0)$$
$$= \frac{\alpha}{\pi} + O(\alpha^2)$$

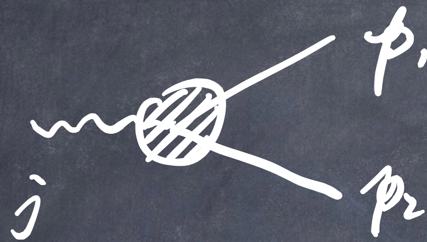
$\sigma \sim$ depend on $\{F_1, F_2\}$

Nuclear form factor: F_1, F_2

↳ characterize shape of particle

diviation from point particle

Sudakov Form factor.



- Sudakov (1954) sum over leading log.

(off-shell)

$$\Gamma(p_1, p_2, q) \sim \exp\left[-e^2 \log\left|\frac{q^2}{p_1^2}\right| \log\left|\frac{q^2}{p_2^2}\right|\right]$$

$$q^2 \gg p_1^2, p_2^2 \gg m^2$$

- Jackiw (1966) $\longrightarrow$ PhD thesis high-energy form factor

(on-shell)

$$\Gamma(q) \sim \exp\left[-e^2 \log^2 \frac{|q^2|}{\mu^2}\right]$$

Further development ~ 1980.

- A. Mueller (79') ~> sub-leading log. resum.
  J. Collins (80') in QED.
- A. Sen (81') → Non-abelian.

→ Factorization.

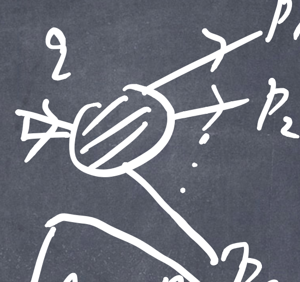
~

~ (soft) × (hard)

→ Sudakov FF.

"4-Loop."

General high-point FF.



F.T.

$$\int d^D x \, e^{-iqx} \langle \underbrace{p_1 \, p_2 \cdots p_n}_{\text{Asmpt. on-shell states.}} | \underbrace{O(x)}_{\text{Local oper.}} | 0 \rangle .$$

$$\boxed{q = \sum_{i=1}^{n} p_i} \qquad \boxed{q^2 \neq 0.}$$

$F_{n,O}(p_1 .. p_n)$

$\langle p_1 .. p_n | 0 \rangle$

Amplitudes.

$\langle O_1 \, O_2 \cdot O_n \rangle$

Correlation funct.

2010 draw attentions.

- Maldacena, Zhiboedov. 2010.
  using AdS/CFT study strong coupling FF.

- Brandhuber et.al. 2010.
  Bork et.al. 2010.

  } weak coupling.

MHV FF:

$$F^{(0),\mathrm{MHV}}_{\mathrm{tr}(\phi^2)}(1^g \ldots i^\phi \ldots j^\phi \ldots n^g) = \frac{\langle ij\rangle^2 \, \delta^4(\sum_i p - q)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

$\underline{\mathrm{tr}(F^2)(1^{g^+} \quad i^{g^-} \quad j^{g^-} \quad n^{g^+})}$ → apply to $\mathcal{N}=4$. QCD

Applications of FF:

- Sudakov FF (IR divergence)
  $\hookrightarrow$ in general gauge theory.
- Anomalous dimensions (UV. renormalization)
- EFT Amplitudes.

$\underline{\underline{H\, tr(F_{\mu\nu}F^{\mu\nu})}}$ + higher dim-vert.

Higgs + gluons Amp $\longleftrightarrow$ Formfactor with $\mathrm{tr}(F^2)$

$$q^2 = m_H^2$$

$\longleftrightarrow$

$F^{(2)}_{\mathrm{tr}(\phi^2)}(123)$ in $N=4$

Brandhuber et.al. 2012.

in QCD.

Gehmann et.al. 2011.

Maximal Trancendental

Principle of Max. Transcendentality.

Lipatov et. al. 2001. $\quad \gamma^{N=4} = \gamma^{QCD}\Big|_{L.T.}$

This talk we focus on

"Lightlike" form factor. in $\mathcal{N}=4$ SYM.

$$F_4^{LL} = \langle 1^g 2^s 3^s 4^g | \mathcal{L}(q) | 0 \rangle$$

$\mathcal{L} \to$ $\mathrm{tr}(F^2)$ + super comp.

$$\boxed{q^2 = 0}$$

$$\boxed{q = \sum_i p_i}$$

- Bootstrap to 2-loops.
- FF OPE.

Bootstrap Method.

$$\text{Loop Amp}/\text{FF} = \sum_i \underline{\underline{\text{Coeff}_i}} \times \underline{\text{Master Integral}_i}$$

physical information.

Theory-independent
Known in many cases.
(once for all).

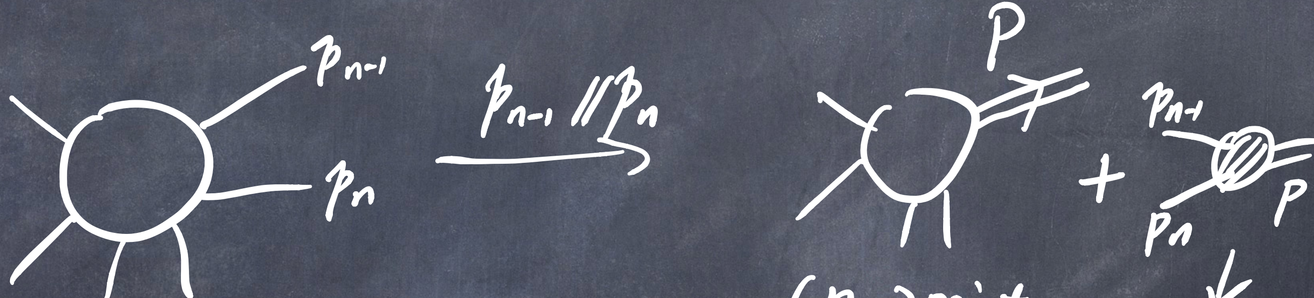
- Traditional method. "Top-down".
- Bootstrap "bottom-up".

Use physical constraints directly to fix final result.

Constraints :

- IR divergence: $\frac{1}{\epsilon^{\#}}$ divergent terms are well-known.

- Collinear limit :



n-point

(n-1) point → Lower point → well-known.

splitting factor. → universal.

BDS straction : Bern, Dixon, Smirnov 2005

$\underline{F^{(1)}}, \underline{F^{(2)}}$

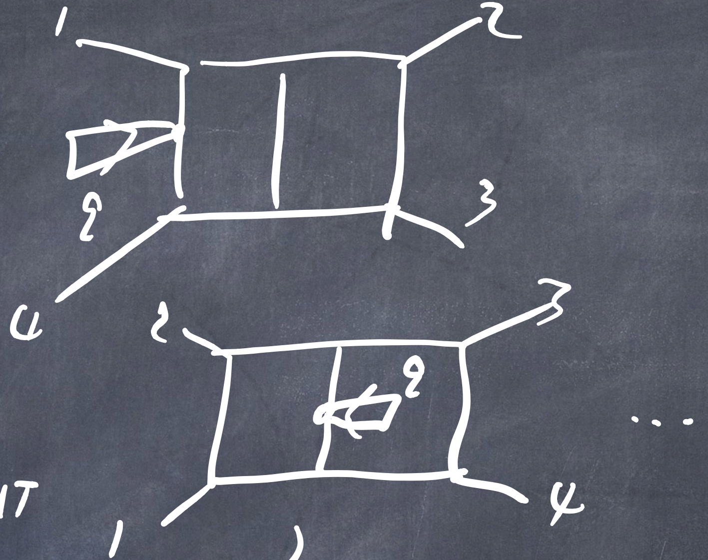
define Remainder function:

$$\boxed{R^{(2)} = F^{(2)}(\epsilon) - \frac{1}{2}\left(F^{(1)}(\epsilon)\right)^2 - f^{(2)}(\epsilon) \cdot F^{(1)}(2\epsilon)}$$

$\underline{f^{(2)}(\epsilon)} = -2\zeta_2 - 2\zeta_3 \epsilon - 2\zeta_4 \epsilon^2 \rightarrow$ contain splitting info.

$R^{(2)}$ ① is finite.

② has trivial collinear limit $R_n^{(2)} \xrightarrow{p_n \parallel p_{n-1}} R_{n-1}^{(2)}$



① Ansatz

$$F_4^{(2)} = \sum_{i=1}^{590} \underbrace{c_i}_{\text{un-known.}} \, \underbrace{I_i^{(2),UT}}_{q^2=0}$$

$q^2=0$ all master integral are known.

② One-loop is Known

$$\frac{\#}{\epsilon^4}+\frac{\#}{\epsilon^3}+\cdots$$

③ Apply constraints.

* Symmetry D_4

$$F_4^{(\ell)} = F_4^{(\ell)}\Big|_{p_i \to p_{i+1}} = F_4^{(\ell)}\Big|_{p_i \to 5-p_i}$$

* IR : $\underline{F_4^{(2)}} \overset{IR}{\sim} \underline{\frac{1}{2}\left(F_4^{(1)}\right)^2 + f^{(2)}_{(\epsilon)} F^{(1)}_{(2\epsilon)}}$

* Collinear.

$$R_4^{(2)} \xrightarrow{p_i \parallel p_{i+1}} \underbrace{R_3^{(2)} = \text{const.}}_{\text{known.}}$$

$\Rightarrow$ We can fix $\underline{\underline{R_4^{(2)}}}$ uniquely!

$p_1, p_2, p_3, p_4, \quad q = \sum_{i=1}^{4} p_i$

S_{ij} : $S_{12}, S_{13}, S_{14}, S_{23}, S_{24}, S_{34}$

$\downarrow$

- $q^2 = 0, \quad \Rightarrow \quad \sum_{i<j} S_{ij} = 0 \Rightarrow$ 5-independent

$\swarrow$

- scale invariance. $u \sim \dfrac{S_{ij}}{S_{12}} \Rightarrow$ 4-independent

$R_4^{(2)}$

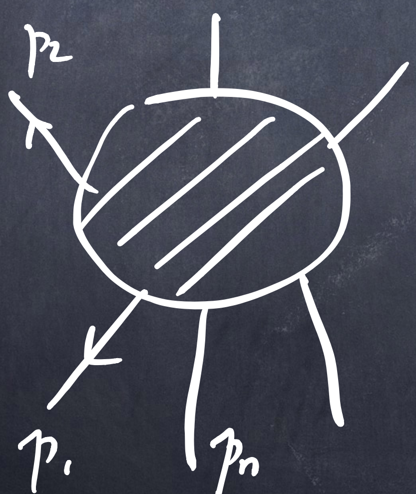
Result shows that there are only

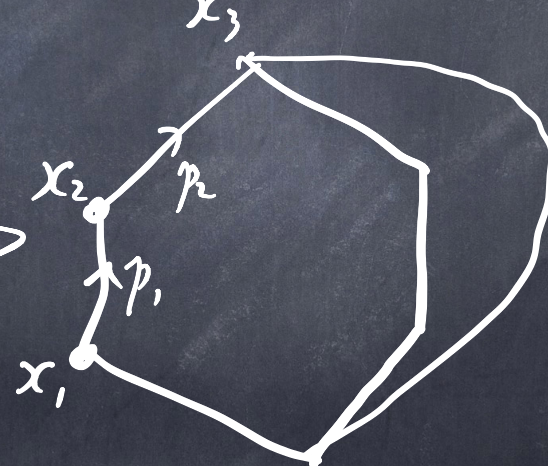
3 independent ratio variables!

⟶ imply there is a hidden symmetry!

Duality between Amp/FF and Wilson Loop.

Alday-Maldacena 07. (strong coupling)





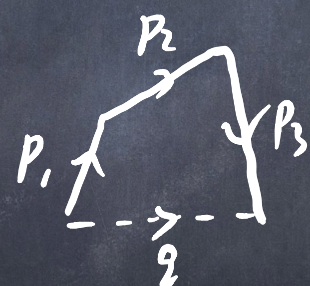
Dual conf sym.
Hidden sym!

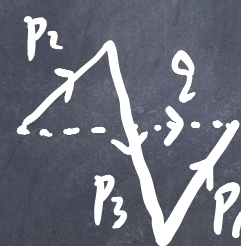
conf sym.

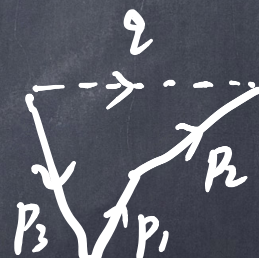
$$\langle W(C)\rangle = \left\langle \mathrm{tr}\, P\, e^{\oint A_\mu(x)\, dx^\mu} \right\rangle$$

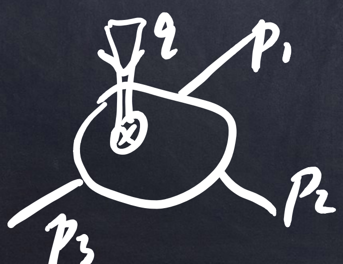
Form factor:

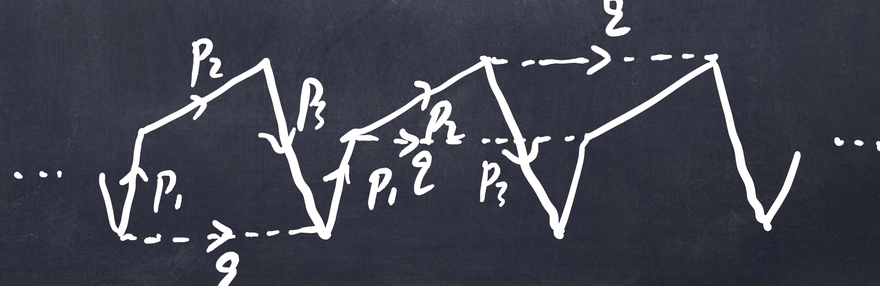
3-pt example:



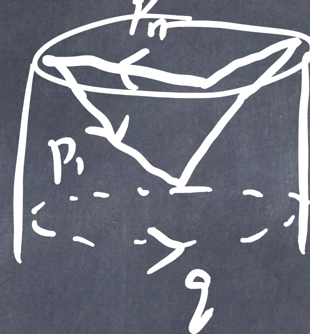








form factor $\leftrightsquigarrow$ periodic Wilson loop



one $q \sim$ period

$\rightarrow$ q direction.

$q^2 \neq 0$ non-lightlike $\xrightarrow{\text{C.T.}}$ curved line

straightline

$q^2 = 0$ Lightlike $\xrightarrow{\text{C.T.}}$ Lightlike.

A dual conformal sym along q-direction:

$$\delta_q x_i^\mu = \frac{1}{2} x_i^2 q^\mu - (x_i \cdot \ell) x_i^\mu$$

$$\delta_q \langle W.L. \rangle = 0 = \underline{\underline{\delta_q R^{(\ell)}}}$$

$\hookrightarrow$ one sym $\rightarrow$ elimiate one variable as mentioned above.

A_n,  W_n : $\underline{3n-15}$

$F_{n,q^2=0}$: $3n-9$

$n=4 : 3\times 4-9 \underline{\underline{=3}}$

$F_{n,q^2\neq 0}$  : $3n-7$

$n=4 : 3\times 4-7=5$

$\{p_1, \dots p_n\}$ $\sum_i p_i = 0$

$p_i^2 = 0$

$3n - \underbrace{15}_{\text{conf group}}$

$10 + 4 + 1$

K^μ

$\{p_1, \dots p_n, q\}$ $p_i^2 = 0$

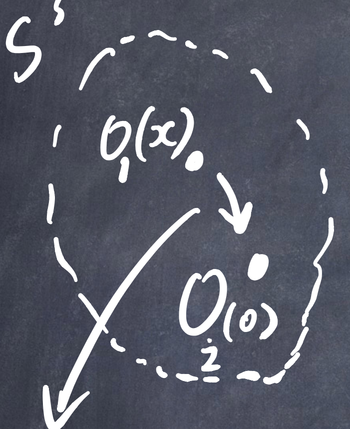
$\sum_i p_i = q$

$3n - (15 - \underset{= P^\mu}{4} - \underset{= K^\mu}{4})$

$= 3n - 7$

OPE

Analogy with usual OPE in CFT.



$$O_1(x)\, O_2(0) \sim \sum \underbrace{C_{12i}}_{\text{structure const.}} \; \underbrace{f(x,\partial_x)}_{\text{determine by C.S. up to } \Delta,\, l.} O_i$$

Dilatation trasformation.

Hamiltonian.

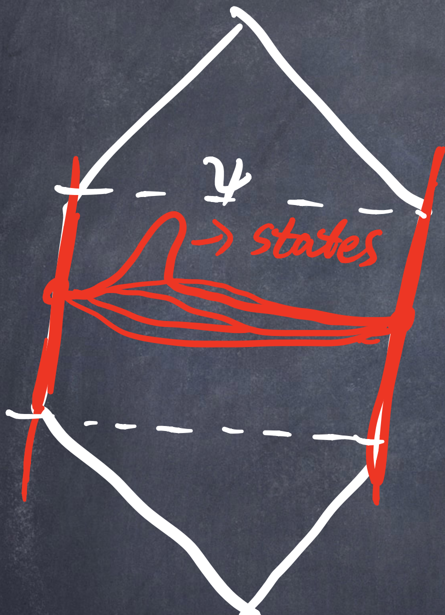
$$\boxed{O_i \text{ characterized by } \Delta,\ l. \qquad D\,O_i = \Delta_i\, O}$$

5 pt correlator $\langle O_1 O_2 \cdots O_5 \rangle$

$$\sim \sum_{\psi_1, \psi_2} C_{12\psi_1} C_{45\psi_2} \cdot \text{conf. P.W.} (\Delta_i)$$

$\{ C_{12\psi}, \Delta_i \}$ CFT data.

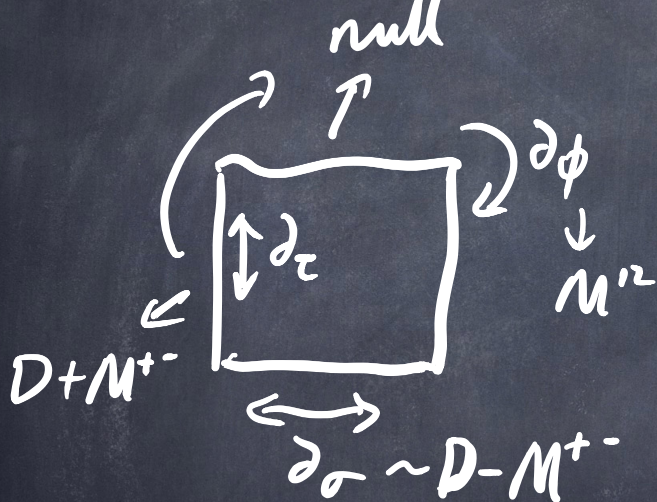
Wilson Loop OPE.



$$W(\text{⬡}) = \sum_{\psi} e^{-E_\psi \tau} \underline{\underline{C_n}}$$

Alday. Gaiotto. Maldacena. Sever. Vieira 2010.

The expansion states are labeld by symetries of a null square



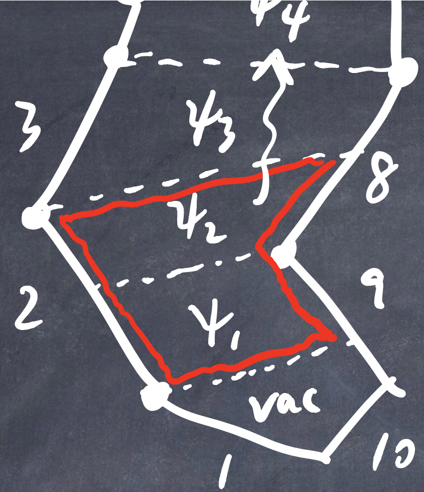
$\partial_\tau, \partial_\sigma, \partial_\phi$ 对易.

$\partial_\phi \sim U(1)$

$\partial_\tau \to E$, $\partial_\sigma \to p$, $\partial_\phi \to m$

$\psi_i \sim \{\sigma_i, \tau_i, \phi_i\}$.

$\downarrow \# = 5$



$$3n-15\Big|_{n=10} = 15 = \underline{\underline{3\times 5}}.$$

$$W_{10} = \sum_{\{\psi_i\}} \prod_{i=1}^{5} e^{-\tau_i E_i + i p_i \sigma_i + i m_i \phi_i}$$

$$\underline{\underline{\underline{P(\text{vac}|\psi_1)\, P(\psi_1|\psi_2) \cdots P(\psi_5|\text{vac})}}}.$$

Analogy :

$\langle \bullet\ \bullet \rangle\ \Delta \longleftrightarrow \square$ null box.

$\langle \cdot \; \cdot \; \cdot \rangle \; \lambda_{ijk} \longleftrightarrow$

pentagon.

$\mathbb{P}(4|4')$

$\sim \mathbb{F}(4)$

$q^2=0$

$\langle : \; : \rangle \; G_4 \longleftrightarrow$

hexagon

$q^2=0$

Form factor.

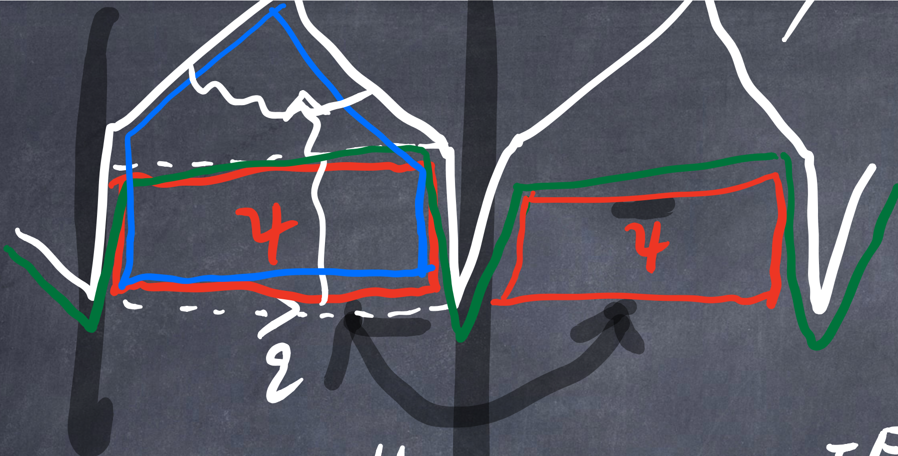
$\underline{3n-9}$ $\quad n=3$ const.

based on

Sever. Tumanov.

Wilhelm 2020.

$q^2 \neq 0$.



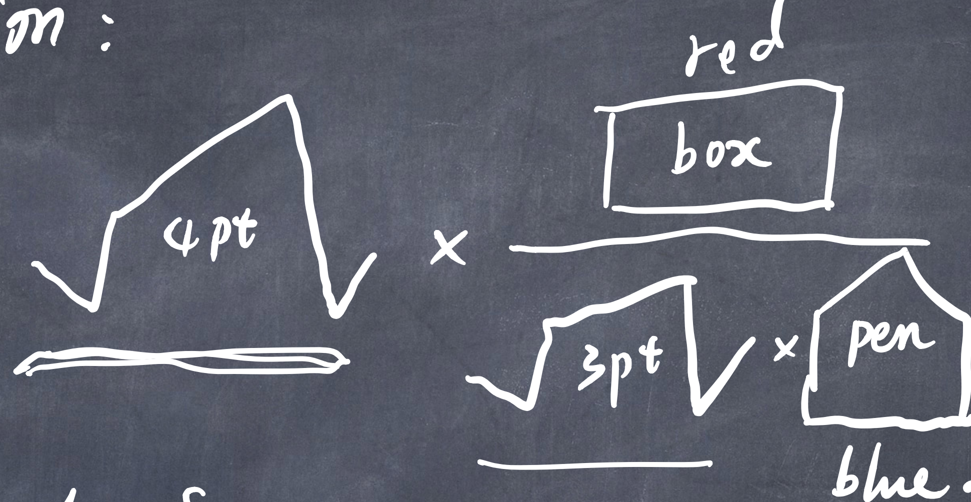
$\psi \sim (\tau, \phi, \sigma)$.

$$\triangle \quad F_4^{LL.} = \sum e^{\underline{-\tau E + i p \sigma + i m \phi}} \, \underline{P(\mathrm{vac}|\psi)} \, \underline{\mathbb{F}(\psi)}$$

New building block for Lightlike FF.

Regularization:

$$W_4^{ll} = F_4^{Reg} =$$



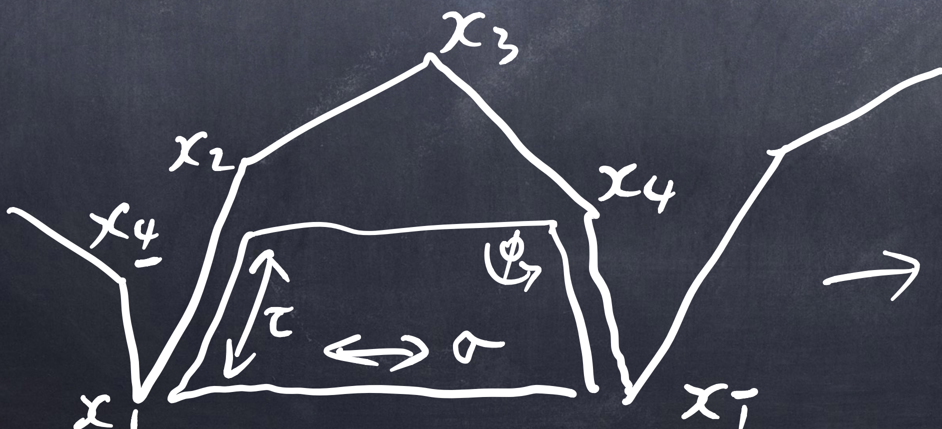
Fuction of $\{\sigma_i, \tau_i, \phi_i\}$.

$$W_4^{ll\,(1)} = -\frac{1}{2}\log\left(\frac{(1-u_1)(1-u_2)}{1-u_3}\right)\log\left(\frac{(1-u_1)(1-u_2)}{1-u_3}\frac{u_3}{u_1}\right)$$

$$u_1 = \frac{s_{12}}{s_{34}} = \frac{x_{43}^2}{x_{3\bar{1}}^2} = e^{-2\sigma}$$

$$u_2 = \frac{s_{23}}{s_{41}} = \frac{x_{24}^2}{x_{4\bar{2}}^2} = e^{-2\tau}$$

$$u_3 = \frac{s_{123}\, s_{341}}{s_{234}\, s_{412}} = \frac{x_{14}^2\, x_{3\bar{2}}^2}{x_{2\bar{1}}^2\, x_{4\bar{3}}^2} = \frac{\cosh(\sigma-\tau) + \cos\phi}{\cosh(\sigma+\tau) + \cos\phi}$$



$\rightarrow \quad \delta_q\, u_i = 0$

From perturbative : Large τ expansion

$$W_4^{\mathcal{U}(1)} = \underline{-2e^{-2\tau}\cos^2\phi} + O(e^{-3\tau})$$

$$W_4^{\mathcal{U}(2)} = 2e^{-\tau}\cos\phi \; h^{(2)}(\sigma)$$

$\downarrow$ Map to OPE.

τ-dependence.

$$W_4^{\mathcal{U}} = 1 + \cos\phi \int \frac{du}{2\pi} e^{-E(u)\tau + iP(u)\sigma} \times \underline{\mathbb{P}(0|u)\mathbb{F}_F(u)} \times \mu_F u$$

(first excitation).

$E(u)$, $P(u)$. $\mathbb{P}(0|u)$, $\mathcal{U}_F(u)$.

△ Know in previous work!

New

→ From perturbative

we can determine.

$$\mathbb{F}_F(u) = g^0 \times 0 + g^2 \times \underline{\underline{(*)}} + O(g^4)$$

$$\downarrow$$

$$-2\,\mathcal{U}_F(u)\big|_{g^2}$$

e $\quad E(u) = \underline{1} + \gamma^{(1)} g^2 + \cdots$

$$W_4^{u(\ell)} = e^{-\tau}\left(\tau^0\,\mathbb{X}_0 + \tau^1\,\mathbb{X}_1 + \cdots + \tau^{\ell-2}\,\mathbb{X}_{\ell-2}\right)$$

$\tau^{\ell-1}\times 0.$

$$\mathbb{X}_{\ell-1} = 0 \quad (\text{from 1-loop})$$

$$\mathbb{X}_{\ell-2} = \int \frac{du}{2\pi}\, e^{2iu\sigma}\left(\frac{(\gamma^{(1)})^{\ell-2}}{(\ell-2)!}\right) \times \left[\mu_F(u)\right]^2_{g^2}$$

$$e^{-\gamma^{(1)}g^2}\,(g^2)^{\ell-2}$$

2 loop data.

This is for any loop order!

Summary & Outlook.

- Amp/FF $\longleftrightarrow$ WL(/)

  momenta. — geometrical 2D.

- Bootstrap + OPE