Conserved charges in the quantum simulation of integrable spin chains

Takuya Okuda

Based on arXiv:2208.00576 (and some extra data)
with K. Maruyoshi, J. Pedersen, R. Suzuki, M. Yamazaki, and Y. Yoshida

Jan. 18, 2023

Plan of the talk

- Motivations and background
- Integrable Trotterization of XXX spin chain
- Quantum gates and circuits
- Quantum devices
- Simulation results/theoretical analysis
- Summary and conclusions

Motivations and background

- Quantum technologies are developing very fast. Governments and companies are making huge investments. Universities and research institutes are hiring many researchers.
- Currently, we only have noisy intermediate-scale quantum (NISQ) devices. Their abilities are limited by noise (errors) and size.
- People hope for early ($\lesssim$ 10 years) practical applications in chemistry (catalyst design) and finance (portfolio optimization).

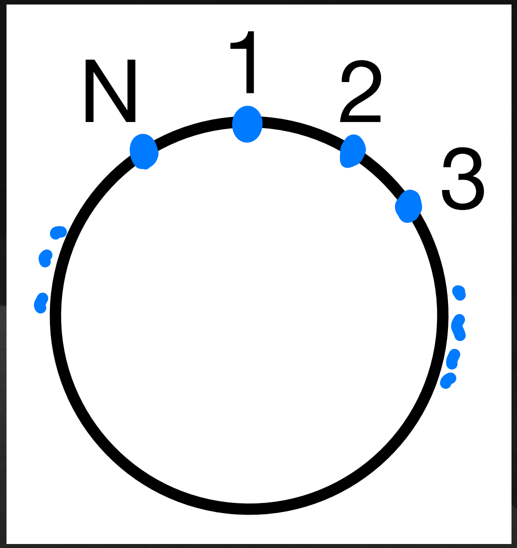
- We expect that the quantum simulation of many-body systems (condensed matter systems and lattice quantum field theories) will be an important target application in the long term ($\gtrsim$ 10 years).
- We wish to **do something interesting** even with the current quantum computers.
  - Q1: Can we quantify the effects of **quantum noise** using a many-body system?
  - Q2: Is there a way to put **discretization errors** under control?

- Because the realistic lattice gauge theories (such as lattice QCD) are hard to simulate on current devices, we consider a spin chain as a toy model.
- It is generally expected that integrable models provide useful benchmarks for quantum simulation because they allow greater analytical control, even when the system size is so large that classical simulation is impossible.
- Today I report the results of our quantum simulation of the Heisenberg spin 1/2 XXX spin chain.

Integrable Trotterization

Integrable Trotterization

[Vanicat, Zadnik & Prosen '17]



- XXX Hamiltonian:

$$H \propto \sum_{j=1}^{N} \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_{j+1} = H_e + H_o \,.$$

$$\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z) \equiv (X, Y, Z)$$

- Ideally, we want to implement e^{-itH}. This is not possible and we resort to the Suzuki-Trotter **approximation**

$$(e^{-i(t/d)H_e} e^{-i(t/d)H_o})^d = e^{-it(H_e+H_o)}(1 + \mathcal{O}(d^{-1}))\,.$$

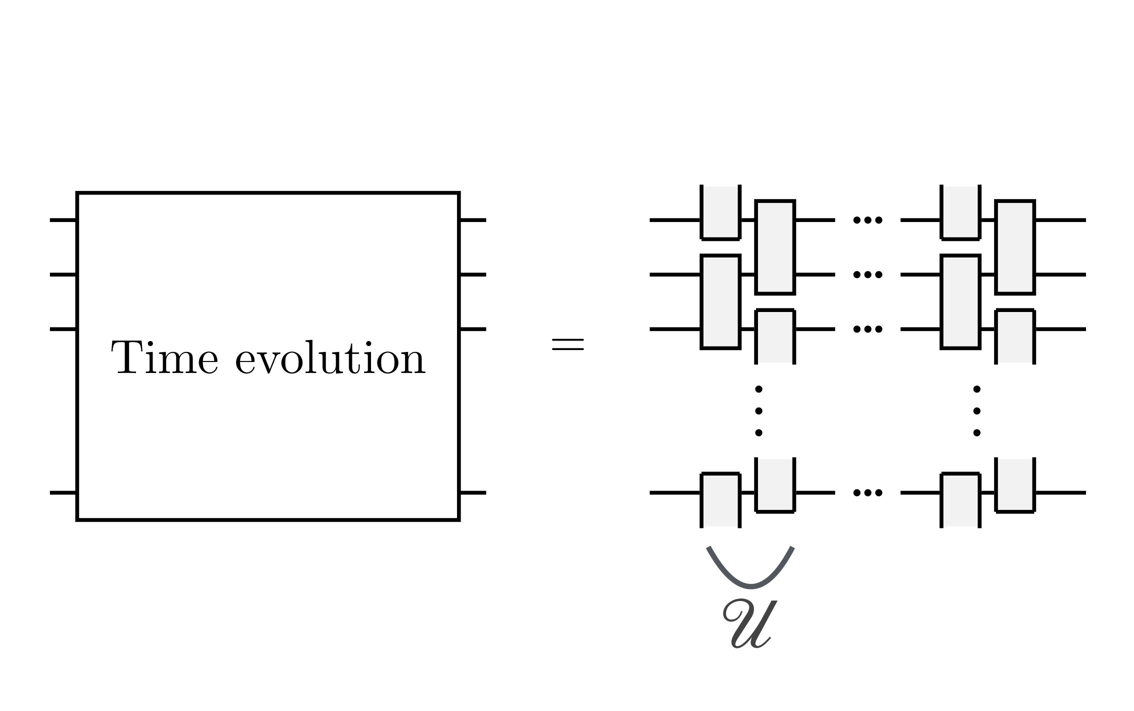
d: number of repetitions ~ depth

The Trotterized small-time evolution (even N, periodic b.c.) can be expressed in terms of the R(-check) matrix:

$$e^{-i\alpha H_e} e^{-i\alpha H_o} = \left(\prod_{j=1}^{N/2} R_{2j-1,2j}(\delta) \right) \left(\prod_{j=1}^{N/2} R_{2j,2j+1}(\delta) \right) =: \mathscr{U}(\delta)\,,$$

$$R_{ij}(\delta) = (1 + i\delta P_{ij})/(1 + i\delta)$$
$$= (\text{phase}) e^{i\alpha(X_i X_j + Y_i Y_j + Z_i Z_j)}\,.$$

$$(\delta = \tan\alpha)$$



Conserved charges

- $\mathscr{U}(\delta)$ commutes with the transfer matrix with specific inhomogeneity

$$T_\delta(\lambda) = \mathrm{tr}_0\left[\overleftarrow{\prod_{1\leq j\leq N}} R_{0j}\big(\lambda - (-1)^j\delta\big)\right] \text{ for any } \lambda \in \mathbb{C}.$$

- Charges $Q_n^{\pm}(\delta) \sim \frac{d^n}{d\lambda^n}\log T_\delta(\lambda)\Big|_{\lambda=\pm\delta/2}$ are **exactly conserved** even with Trotterization, which give rise to time-discretization errors for other observables.

- $Q_n^{\mathrm{dif}} \equiv [Q_n^+(\delta) - Q_n^-(\delta)]/\delta$ is also conserved.

- Densities $q^{[n,\pm]}_{j,j+1,\ldots,j+2n}$ in higher charges

$$Q_n^+(\delta) = \sum_{j=1}^{N/2} q^{[n,+]}_{2j-2,2j-1,\ldots,2j+2n-2}(\delta)\,,$$

$$Q_n^-(\delta) = \sum_{j=1}^{N/2} q^{[n,-]}_{2j-1,2j,\ldots,2j+2n-1}(\delta)$$

can be computed via the recursion relation $Q^\pm_{n+1} \sim [B, Q^\pm_n]$ on an infinite chain, where B is a discrete (Lorentz) boost transformation. [Vanicat et al.]

- We implemented the recursion in Mathematica programs.

known

$$q^{[1,\pm]}_{1,2,3}(\delta) = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 \mp \delta \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3) + \delta^2 \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 \,,$$

$$\begin{aligned} q^{[2,\pm]}_{1,2,3,4,5}(\delta) = &\mp 2\delta(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_4 + \boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_5 - \boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_5) - (1-\delta^2)\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) - \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) - \delta^2 \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) \\ &- \delta^2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) - \delta^4 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) \pm \delta \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \pm \delta \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) \\ &\pm \delta^3 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \pm \delta^3 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) - \delta^2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \,, \end{aligned}$$

$$\begin{aligned} q^{[3,+]}_{1,2,3,4,5,6,7} = &-4\boldsymbol{\sigma}_6 \cdot \boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_7 - 4\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) \\ &+\delta\Big(10\boldsymbol{\sigma}_5 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 4\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) + 8\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) - 4\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) \\ &-2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_6) - 4\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6)\Big) \\ &+\delta^2\Big(2\boldsymbol{\sigma}_6 \cdot \boldsymbol{\sigma}_7 - 10\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_6 \\ &-6\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 6\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 6\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) \\ &-6\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) + 2\boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) \\ &+2\boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6)\Big) \\ &+\delta^3\Big(6\boldsymbol{\sigma}_5 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 4\boldsymbol{\sigma}_4 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 8\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) \\ &-2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_6) + 4\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) - 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_7) + 4\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) \\ &-2\boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) \\ &-2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) - 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7)\Big) \\ &+\delta^4\Big(-2\boldsymbol{\sigma}_6 \cdot \boldsymbol{\sigma}_7 - 8\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_7 - 2\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_7 - 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) \\ &+2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6) \\ &+2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7)\Big) \\ &+\delta^5\Big(4\boldsymbol{\sigma}_5 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6 \times \boldsymbol{\sigma}_7) \\ &-2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7)\Big) \\ &+\delta^6\Big(-4\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_7)\Big)\,. \end{aligned}$$

new

Densities and charges are traceless.

Here $\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \cdots \times \boldsymbol{\sigma}_{\ell-1} \times \boldsymbol{\sigma}_\ell) := \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times (\boldsymbol{\sigma}_3 \times (\cdots \times (\boldsymbol{\sigma}_{\ell-1} \times \boldsymbol{\sigma}_\ell) \cdots)))$

Basics of quantum gates and circuits

Qubits

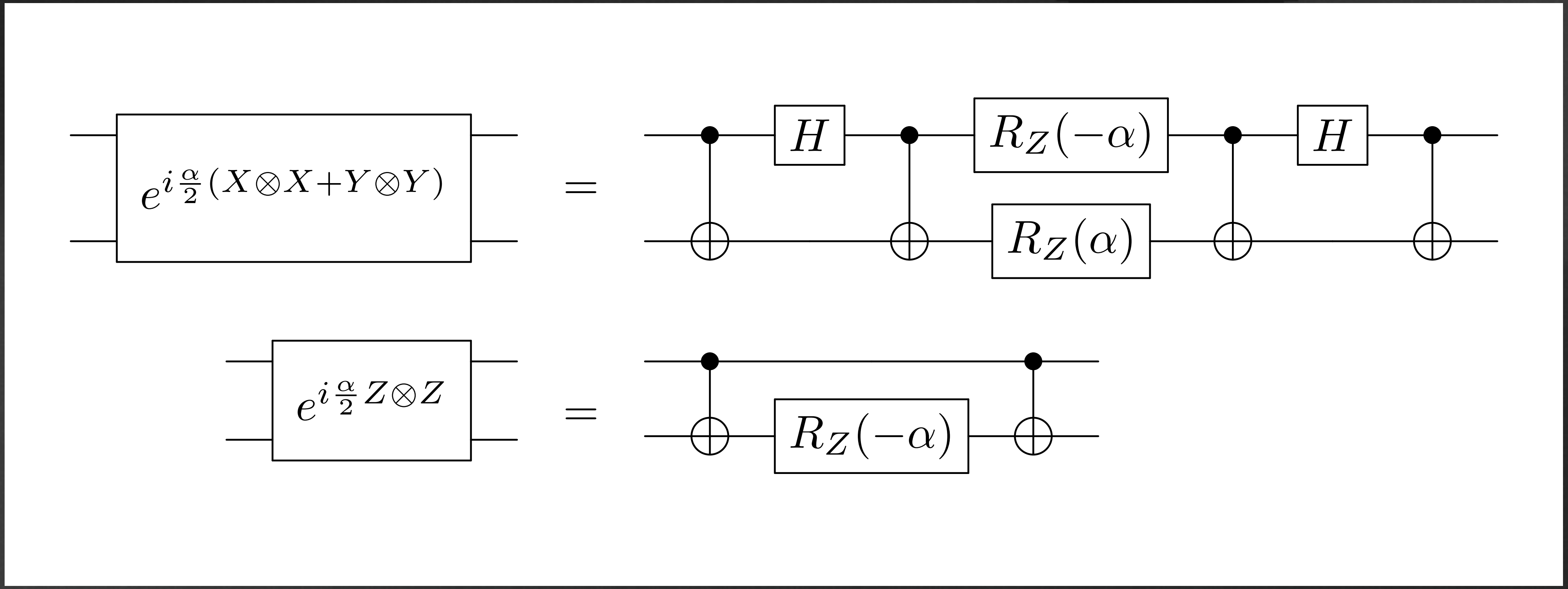
- A quantum computer comes with qubits.
- Each qubit is a 2-dimensional complex vector space spanned by the eigenstates $|0\rangle$ and $|1\rangle$ of Z with eigenvalues 1 and -1.

Quantum gates

Gates are unitary transformations that act on a single qubit or multiplet qubits. We use

- Pauli gates $X, Y,$ and Z,
- Z-rotation $R_Z(\alpha) = e^{-i(\alpha/2)Z}$,
- Hadamard gate $H = 2^{-1/2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $X \leftrightarrow Z$,
- Phase gate $S = \mathrm{diag}(1,i)$, $X \leftrightarrow Y$,
- "controlled X" = CX =$CNOT$: $CX_{12}\,|\,s_1, s_2\rangle = (X_2)^{s_1}\,|\,s_1, s_2\rangle$ ($s_1, s_2 \in \{0,1\}$).

The R matrix $R_{ij}(\delta) = (\text{phase})e^{i\alpha(X_iX_j+Y_iY_j+Z_iZ_j)}$ can be implemented by elementary gates.



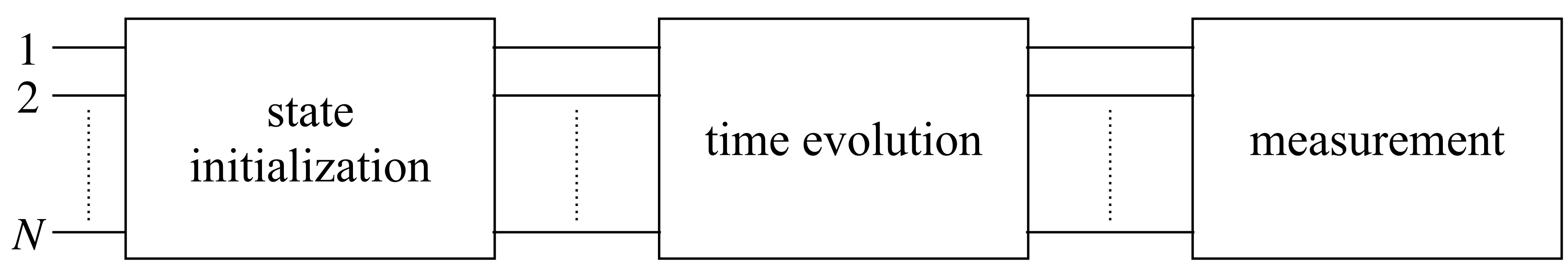
Quantum circuit

- Initialization: by default the quantum device prepares $|00\ldots0\rangle$. We further apply some of H, S, and Pauli gates (X, Y, and Z) to prepare a simultaneous eigenstate $|s_1\ldots s_N\rangle_{P_1\ldots P_N}$ of $P_i \in \{X, Y, Z\}$ with eigenvalues $(-1)^{s_i}$.

- Time evolution: d repetitions of

$$\mathscr{U}(\delta) = \left(\prod_{j=1}^{N/2} R_{2j-1,2j}(\delta)\right)\left(\prod_{j=1}^{N/2} R_{2j,2j+1}(\delta)\right).$$

- Measurement: we measure the eigenvalue of X, Y, or Z for each qubit.



Estimating observables

- One can compute (estimate) the expectation value of a charge

$$Q = \sum_{P \in \{I,X,Y,Z\}^{\otimes N}} c_{Q,P} P, \quad c_{Q,P} \in \mathbb{C},$$

  from the measurement results in various measurement bases.

- See our paper for concrete formulas (no originality claimed) for the expectation value and the statistical uncertainty.

Quantum devices

IBM: Superconducting devices

- IBM uses superconducting transmon qubits. These are made of materials such as niobium and aluminum placed on a silicon chip. Two energy-levels form an approximate qubit.

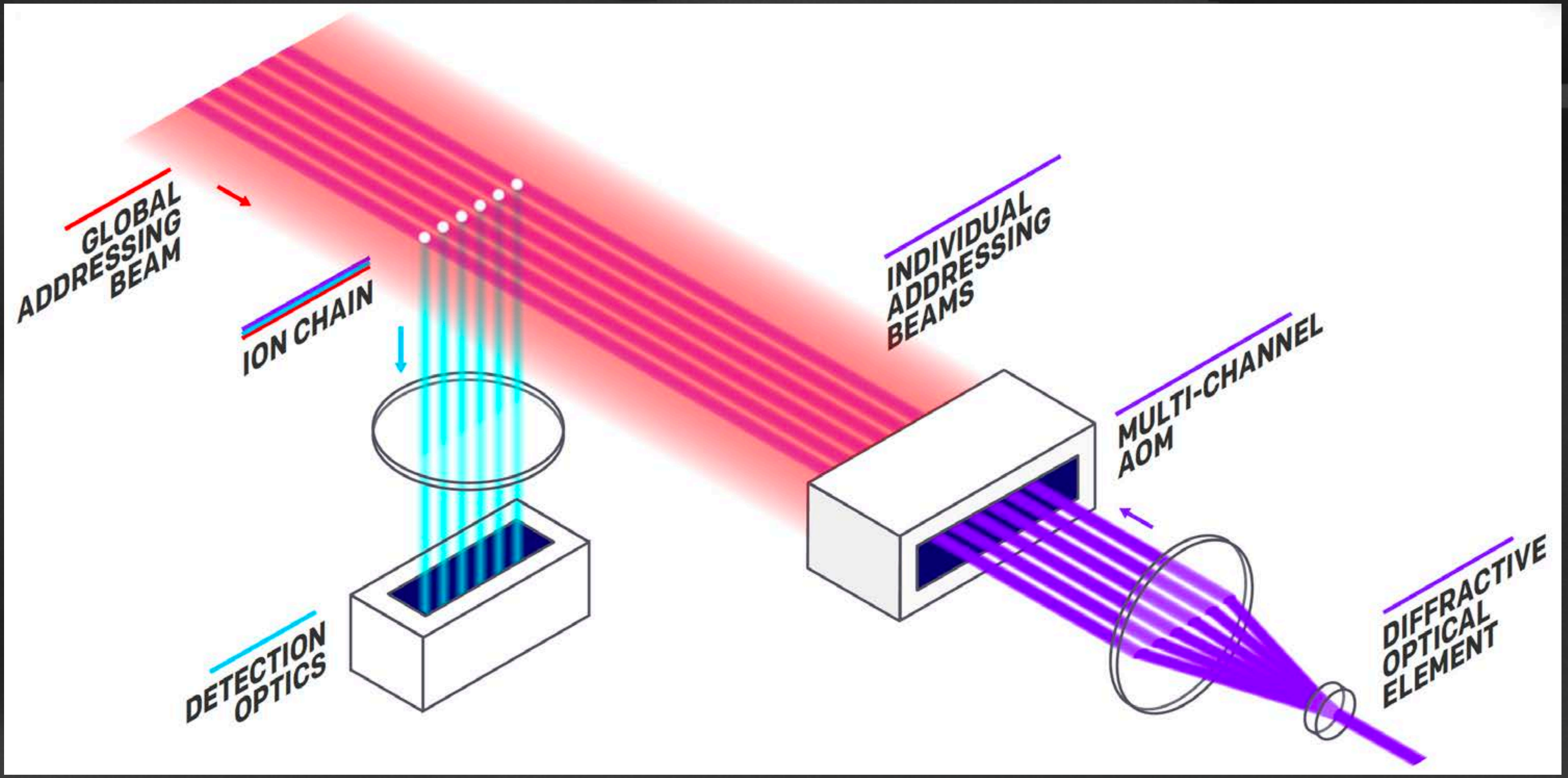
- We obtained access to the devices through the University of Tokyo. (Supported by UTokyo Quantum Initiative).

- We used the ibm_kawasaki and ibm_washington processors.

source: https://japan.zdnet.com/article/35174399/

IonQ: trapped ion devices

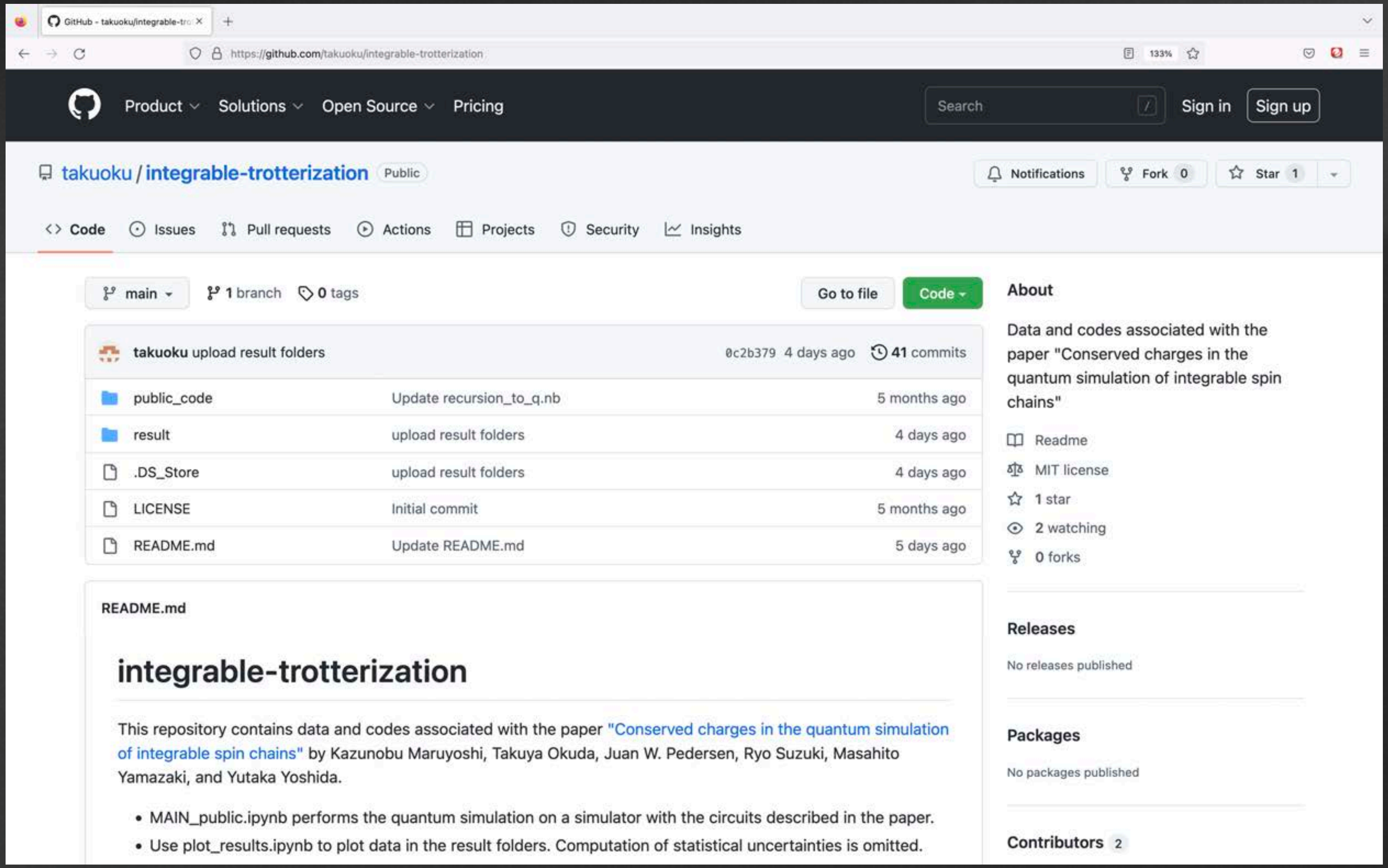
- We mainly used IonQ's device called Harmony. (Not in the current version of the e-print.)
- A linear chain of $^{171}\mathrm{Yb}^{+}$ ions near an electrode trap.
- 11 qubits with all-to-all couplings.
- We got indirect access through Google Cloud and direct access through IonQ itself.



arXiv:1903.08181

Cloud access via Qiskit

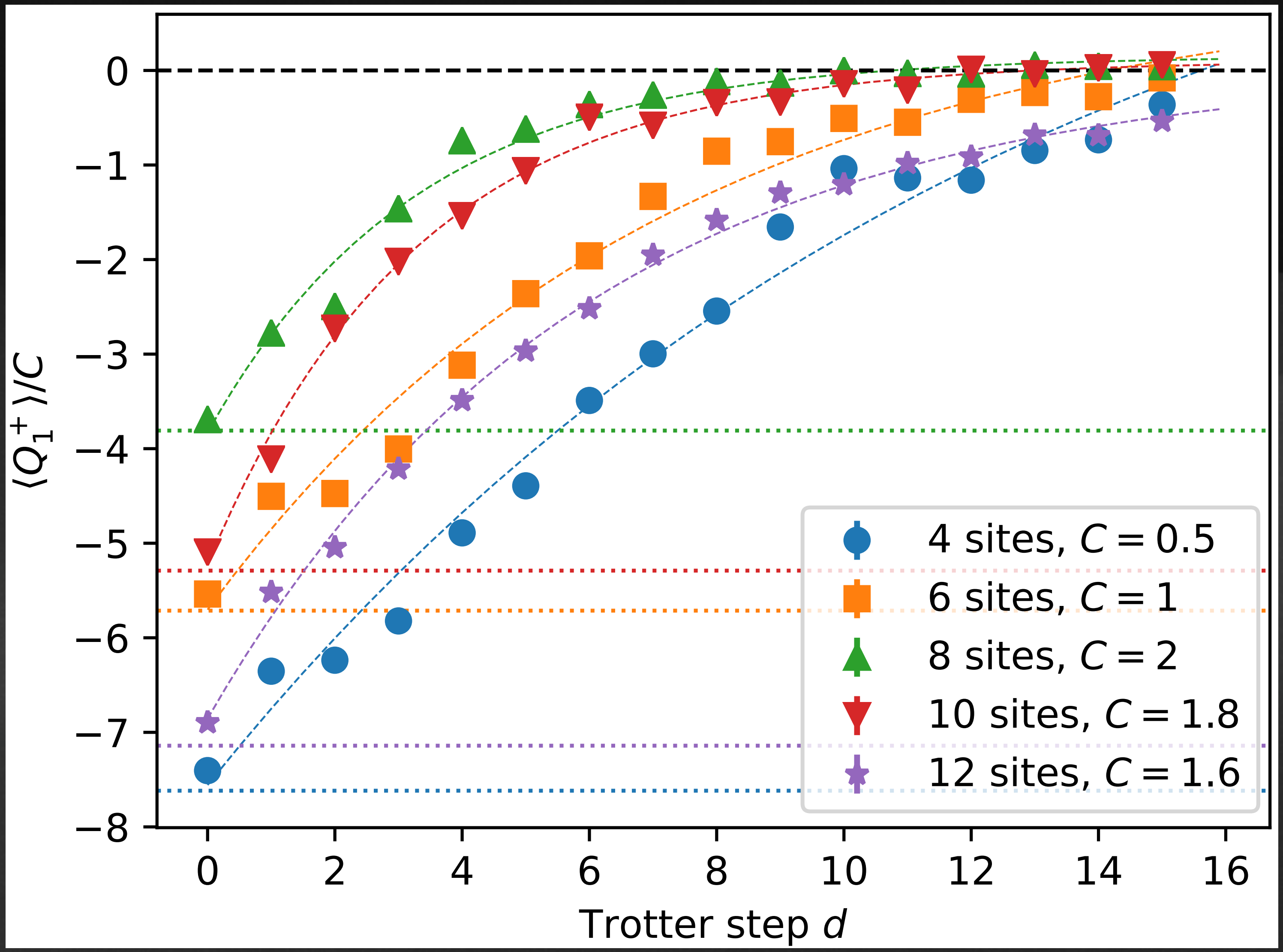
- We used the SDK called Qiskit to control the IBM and IonQ quantum computers.
- Our (mostly Python) programs are available via a GitHub repository.



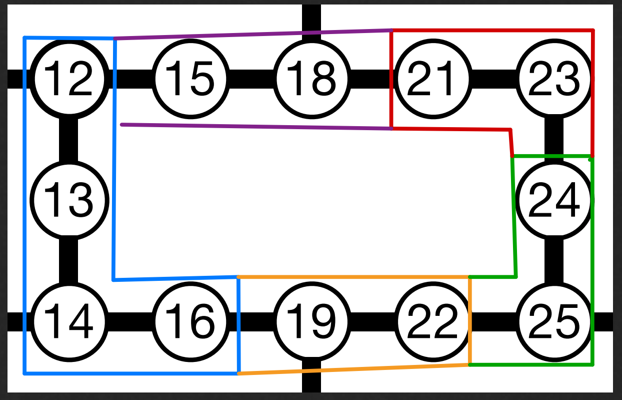
Results of real-device simulations

Simulation results for ibm_kawasaki

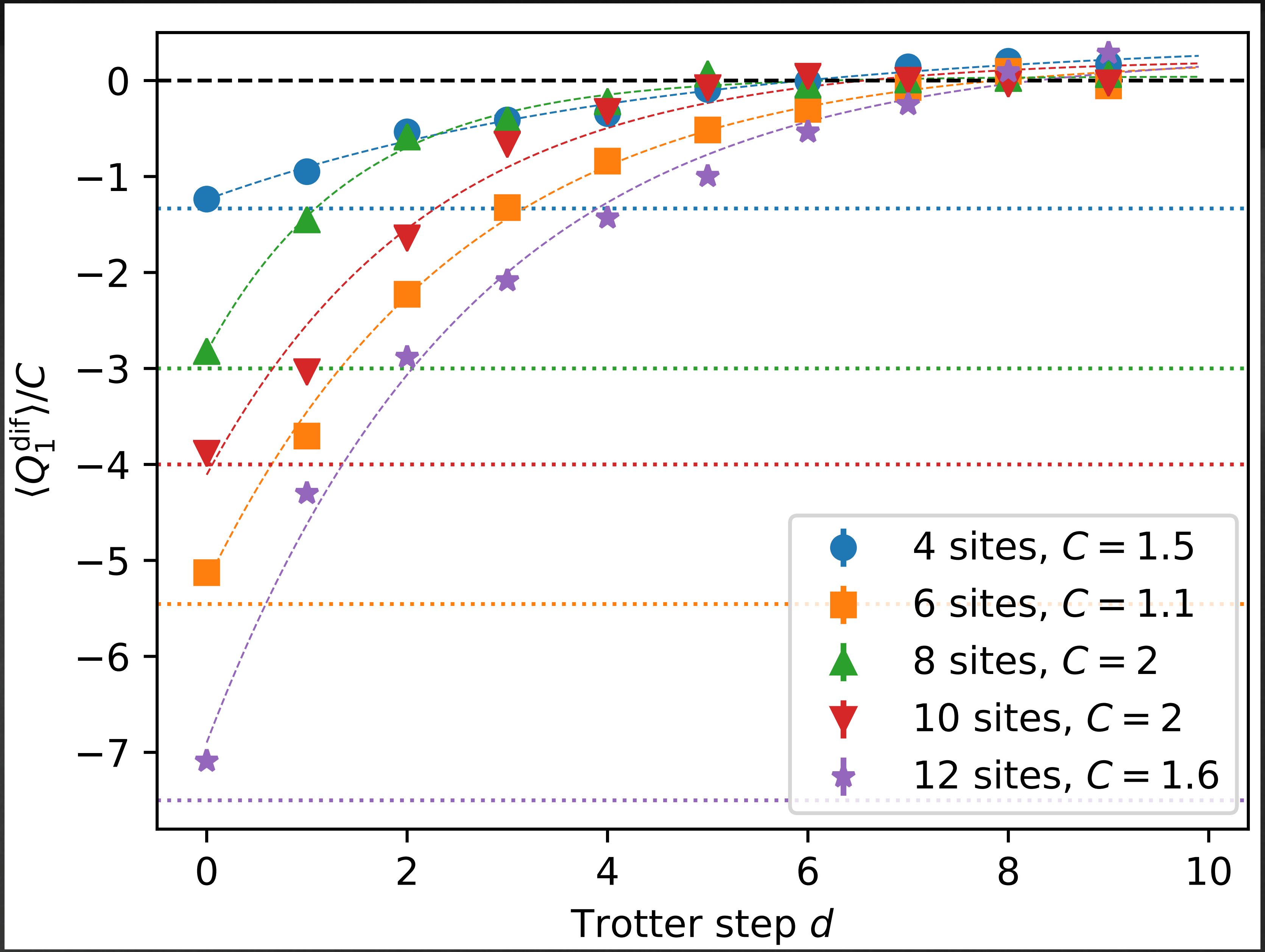
- $\langle Q_1^+ \rangle = \mathrm{tr}(\rho Q_1^+)$ **decays exponentially** to zero asymptotically, **due to noise.** No error mitigation.
- Error bars are hidden by markers. Rescaled for better visibility. The theoretical values are shown by dotted lines. Fit by $c_1 e^{-\gamma d} + c_2$.
- The initial state is $|0101\ldots01\rangle$.
- Large fluctuations from one step to the next. (Due to change in device parameters?)



Only the 12-site simulation is for a circular topology.

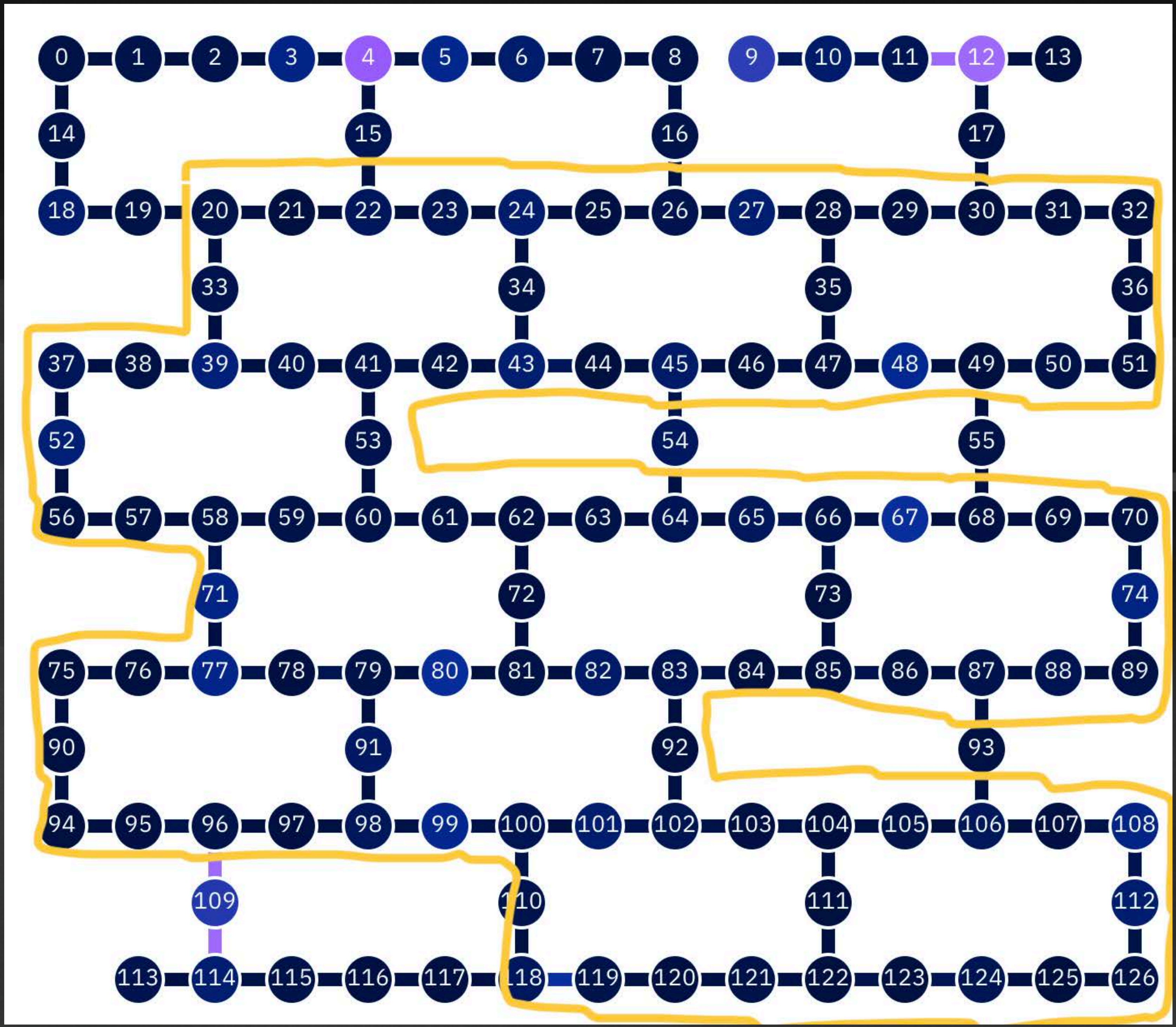


- Similar results for $Q_1^{\rm dif} = [Q_1^+(\delta) - Q_1^-(\delta)]/2$.
- The initial states are chosen appropriately to give non-zero theoretical expectation values.



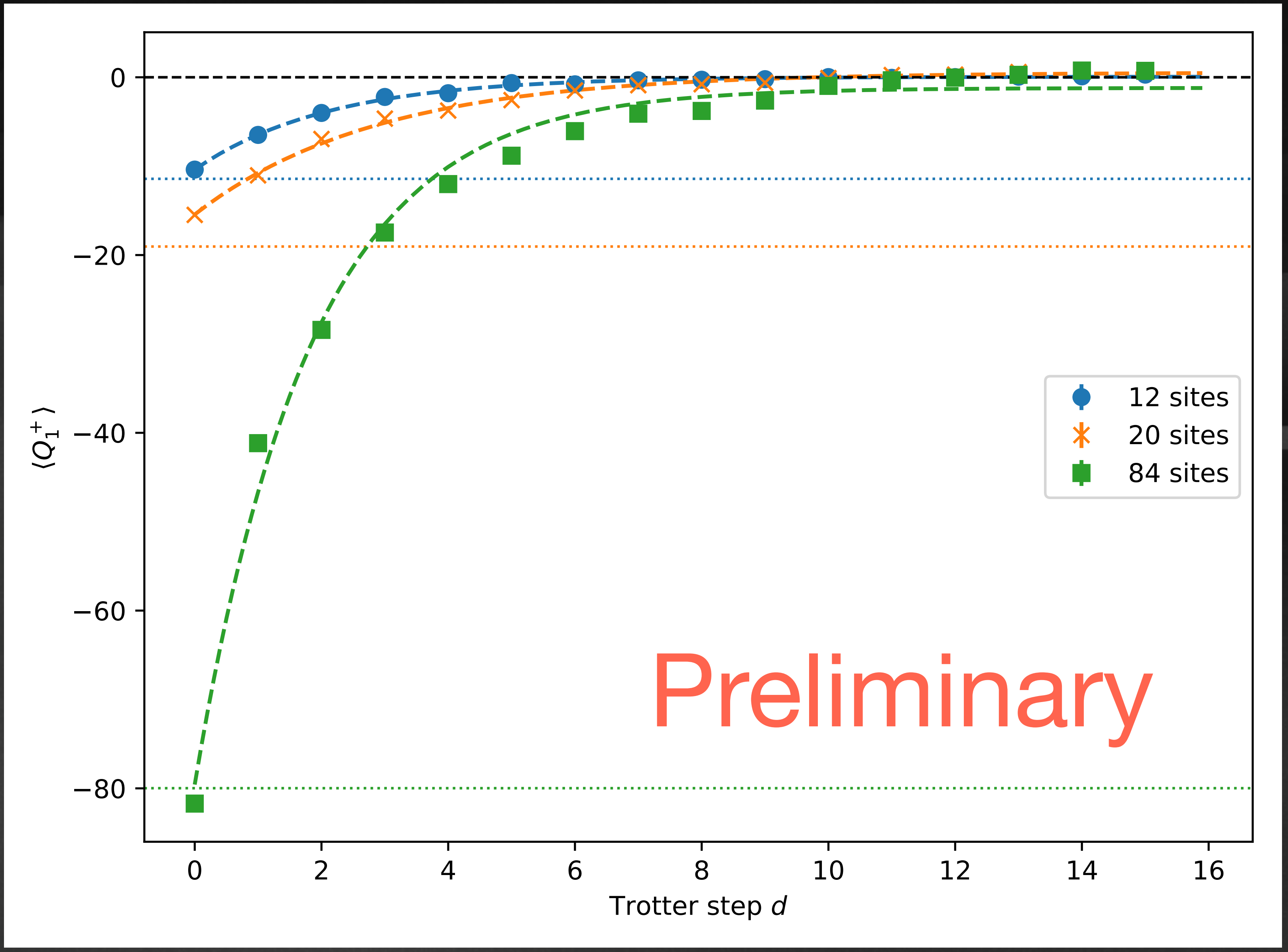
Simulations on a 127-qubit IBM device

- Quantum device ibm_washington with 127 qubits.
- We ran simulations with qubits on loops of size 12, 20, and 84. The 84-qubit loop is shown in the figure.
- To have slower decays, it is important to avoid faulty (purple) qubits and connections.



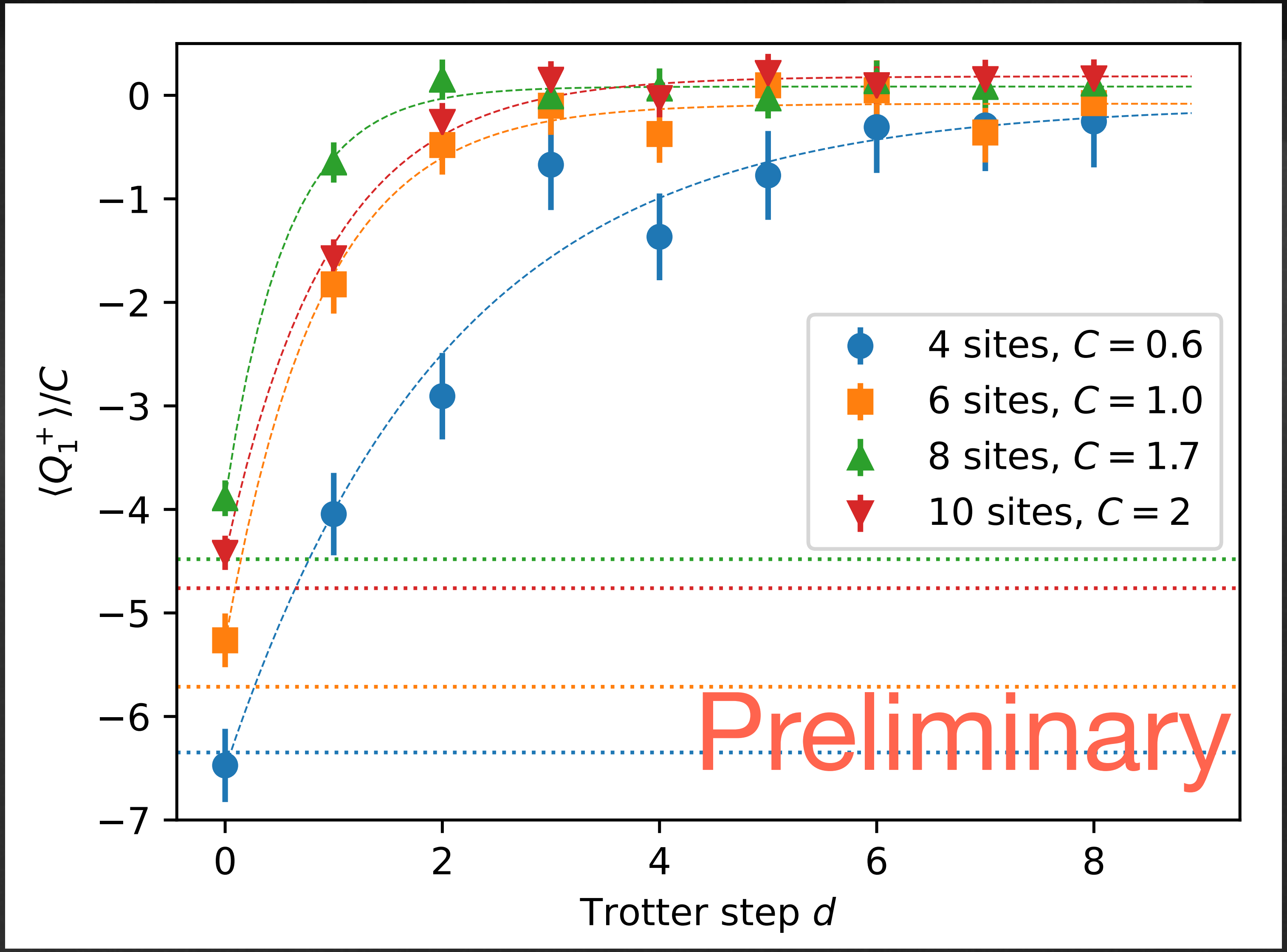
Simulation results on large chains

- Loops of size 12, 20, and 84.
- Similar exponential decays of $\langle Q_1^+ \rangle$.
- For the 84-site run, we had 10^6 shots (circuit executions) for each value of d.
- (There were significant time gaps between some data.)
- (Not in the current version of the e-print.)



Simulation results for IonQ Harmony

- Similar exponential decays.
- To have slower decays, it seems important to use the qubits (ions) in the middle of the linear chain.



Simulator results and theoretical analysis

Numerical noise models

- We ran digital quantum simulations on the Qiskit (classical) simulator with noise models.
- We considered two noise models:

1. (1-qubit) depolarizing error channels inserted after 1- and 2-qubit gate operations.
2. (1-qubit) amplitude-and-phase damping error channels inserted after 1- and 2-qubit gate operations.

Classical emulation of quantum simulation with a depolarizing noise model

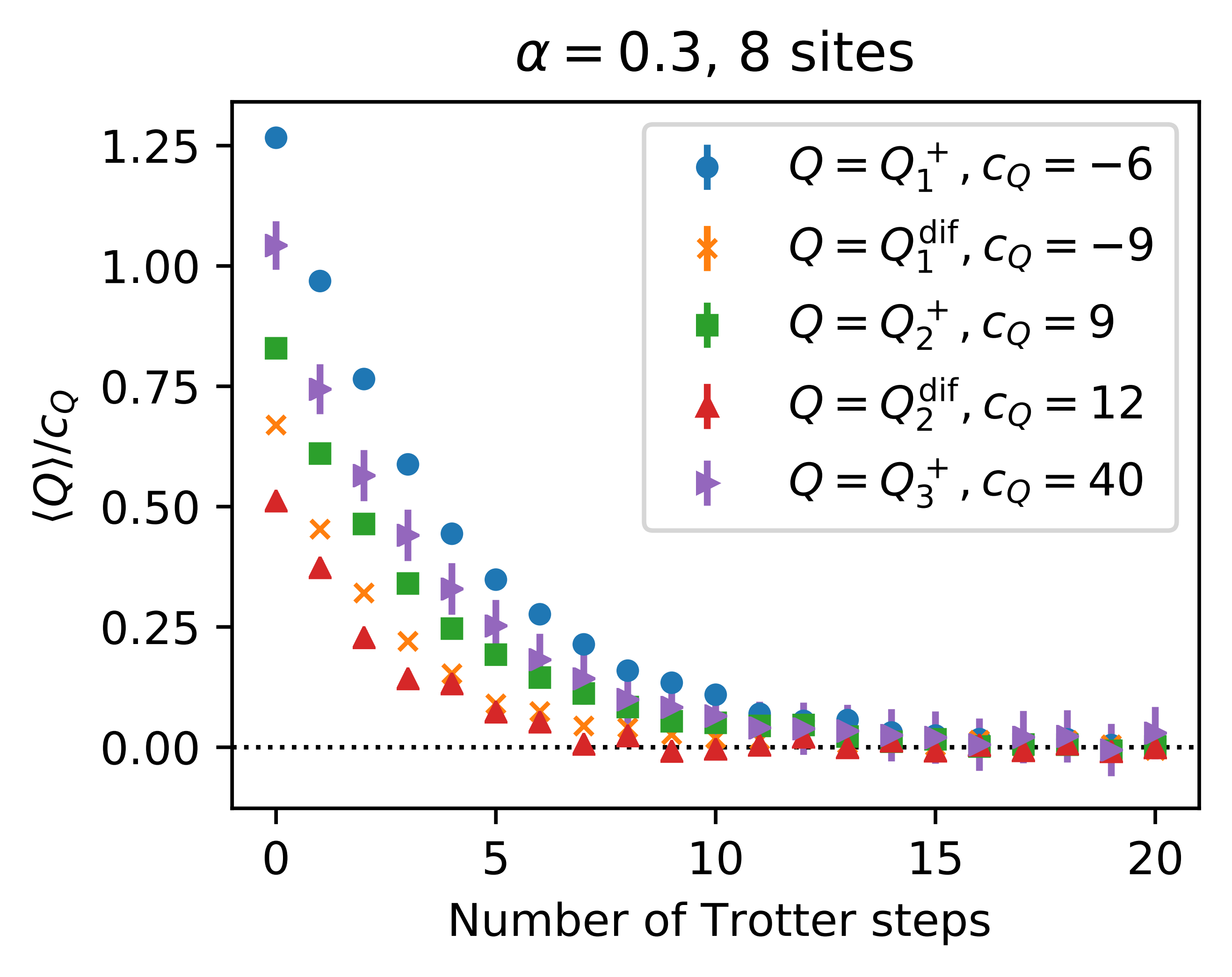
- $\Phi_{\text{depo}}(\rho) = \sum_{j=1}^{4} D_j \rho D_j^\dagger$ with

  $$D_1 = \sqrt{1 - \frac{3p}{4}}\, I, \quad D_2 = \sqrt{\frac{p}{4}}\, X,$$

  $$D_3 = \sqrt{\frac{p}{4}}\, Y, \quad D_4 = \sqrt{\frac{p}{4}}\, Z$$

  inserted after gate operations.

- $\langle Q_j^+ \rangle$ and $\langle Q_j^{\text{dif}} \rangle$ decay exponentially to zero. This suggests that the finite state is completely mixed.



Classical emulation of quantum simulation with a amplitude-and-phase damping noise model

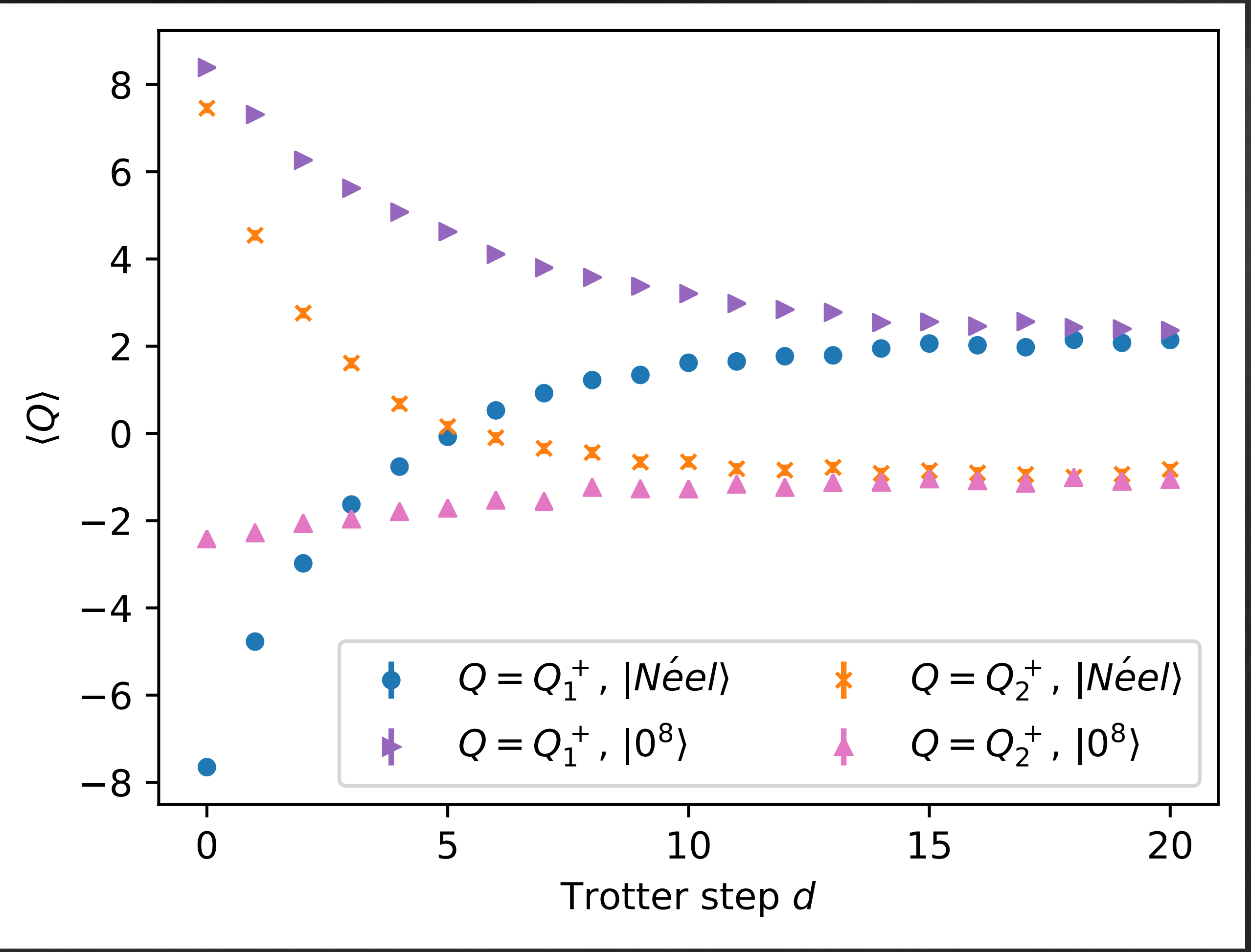
- $\Phi_{\text{damp}}(\rho) = \sum_{j=1}^{3} D_j \rho D_j^\dagger$ with

$$D_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda_a-\lambda_p} \end{pmatrix},$$

$$D_2 = \begin{pmatrix} 0 & \sqrt{\lambda_a} \\ 0 & 0 \end{pmatrix}, \quad D_3 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\lambda_p} \end{pmatrix}$$

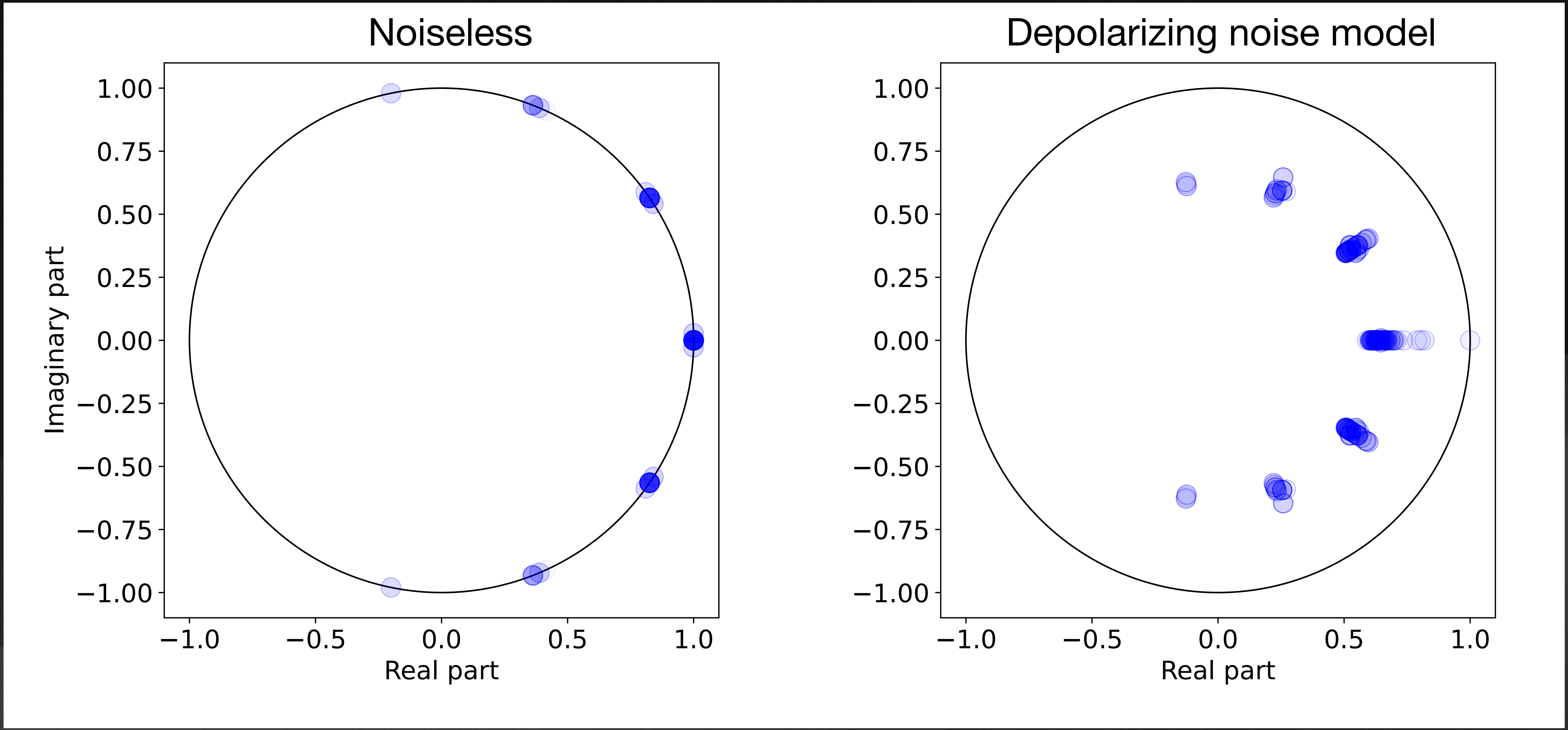
inserted after gate operations.

- $\langle Q_j^+ \rangle$ (and $\langle Q_j^{\text{dif}} \rangle$) asymptote to finite values. The finite state is unique and is NOT completely mixed. Checked by quantum tomography.



Analysis fo quantum channels

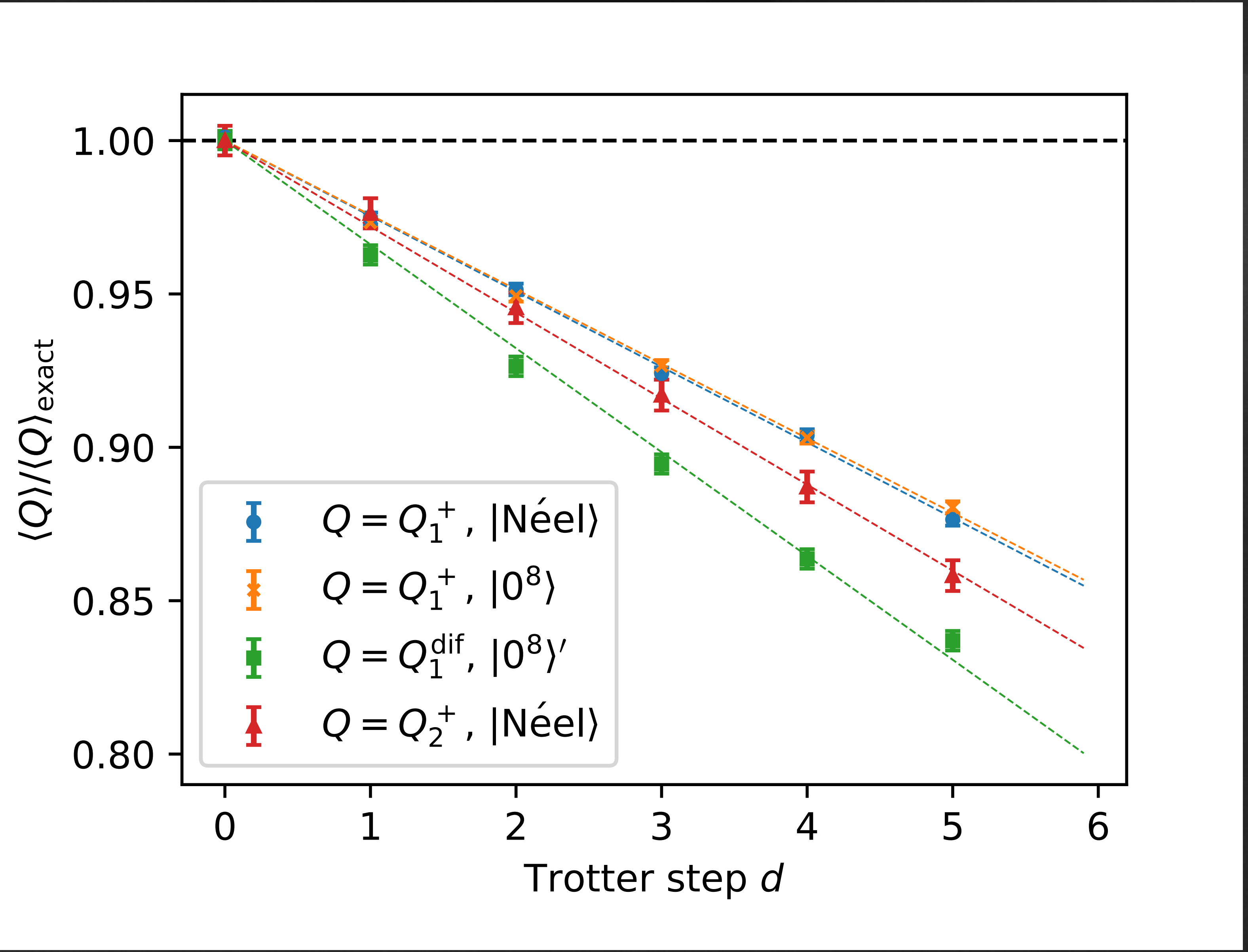
- The initial state $\rho_0 = |\psi_0\rangle\langle\psi_0|$ is mapped, at Trotter step d, to $\Phi^d(\rho)$, where Φ is a noisy time evolution for a single step.
- The expectation value of a conserved charge Q at step d is $\langle Q\rangle_d = \mathrm{tr}[\Phi^d(\rho)Q]$.
- We studied the eigenvalue distribution of the linear map $\rho \to \Phi(\rho)$.



- The eigenvalues for the single time step Φ on 4 sites.
- In the noiseless case, the evolution is unitary and the eigenvalues are on a unit circle.
- In the depolarizing noise model, all the eigenvalues except one are strictly inside the unit circle. There remains a single eigenvalue 1, corresponding to the unique fixed point (completely mixed state) of Φ.

Possible use of conserved charges as benchmarks for future quantum computing

- For future quantum devices we expect smaller error rates. We propose to use the higher conserved charges of the integrable Trotterization as benchmarks.
- On a classical simulator, we numerically computed the time evolution on 8 sites.
- The slopes of early-tiime decays depend on the types and the degrees of the charges.



Summary and conclusions

- We implemented the integrable Trotterization of the Heisenberg spin 1/2 XXX spin chain on real quantum computers and on classical simulators. We used superconducting devices of IBM and trapped ion devices of IonQ.
- As expected, conserved charges decay due to noise on the current quantum devices.
- The early-time decay rate seems to depend on the type and the degree of the charge. Higher charges are candidates of benchmarks for the future quantum simulation.