Emergent symmetry and free energy

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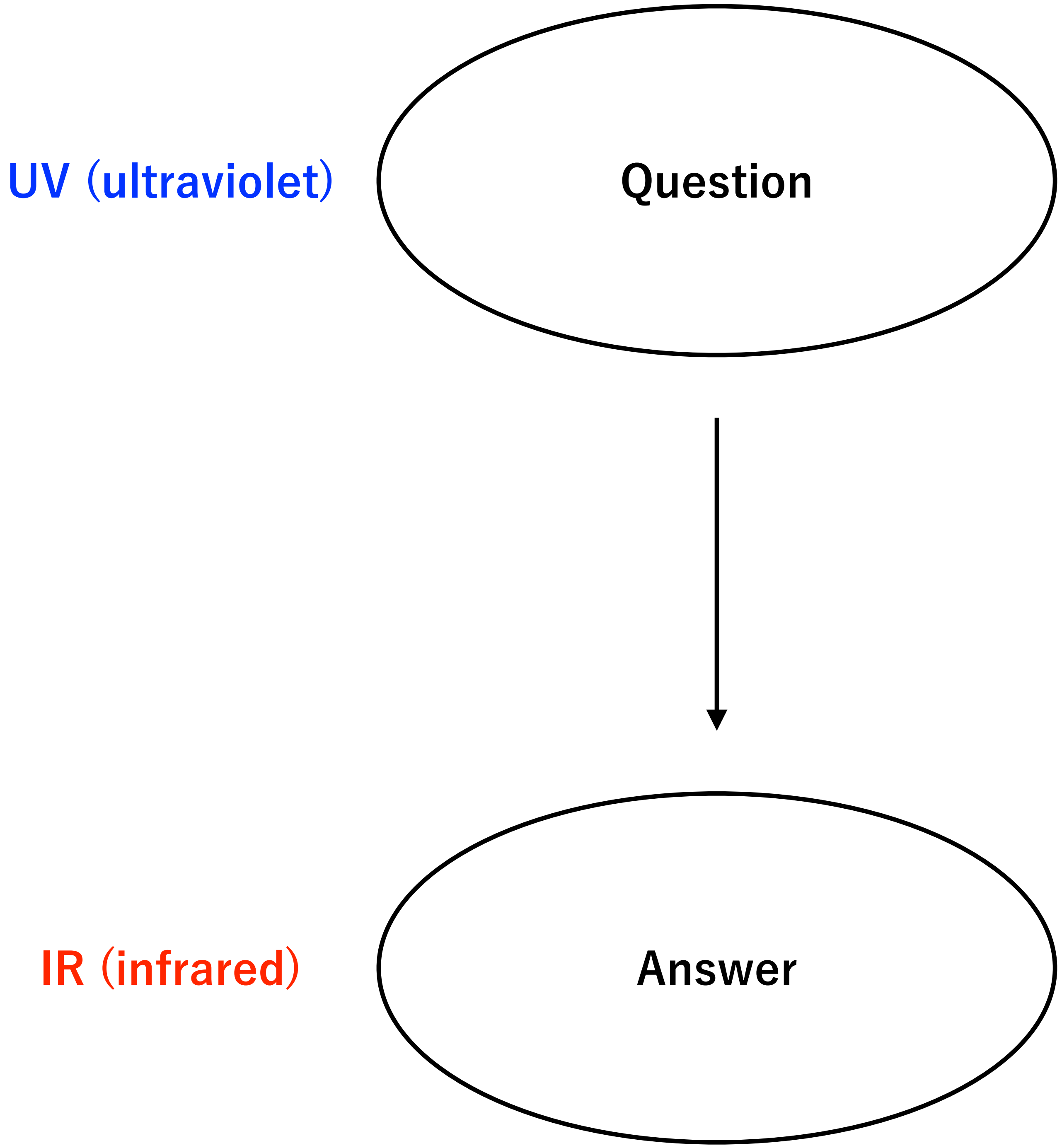
YMSC

Based on

2109.02672 (KK), 2207.06433 (KK), 2207.10095 (KK), 2209.00016 (KK)

Renormalization Group (RG)



Example: QCD

- We have known UV description for decades;
  $SU(3)$ gauge theory w/ fundamental quarks.

- Even perturbative computation is possible.

- But we haven't succeeded to show its IR behaviors:
  - Spont. breaking of chiral sym. (for massless quarks)
  - Confinement

Possible answers

Symmetry\Gap	Gapped (or TQFT)	Gapless (~CFT)
Preserved		
Spont. broken		

SSB and free energy

SSB can be understood as minimization of free energy

$$F = E - TS\,.$$

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$$F = E - TS\,.$$

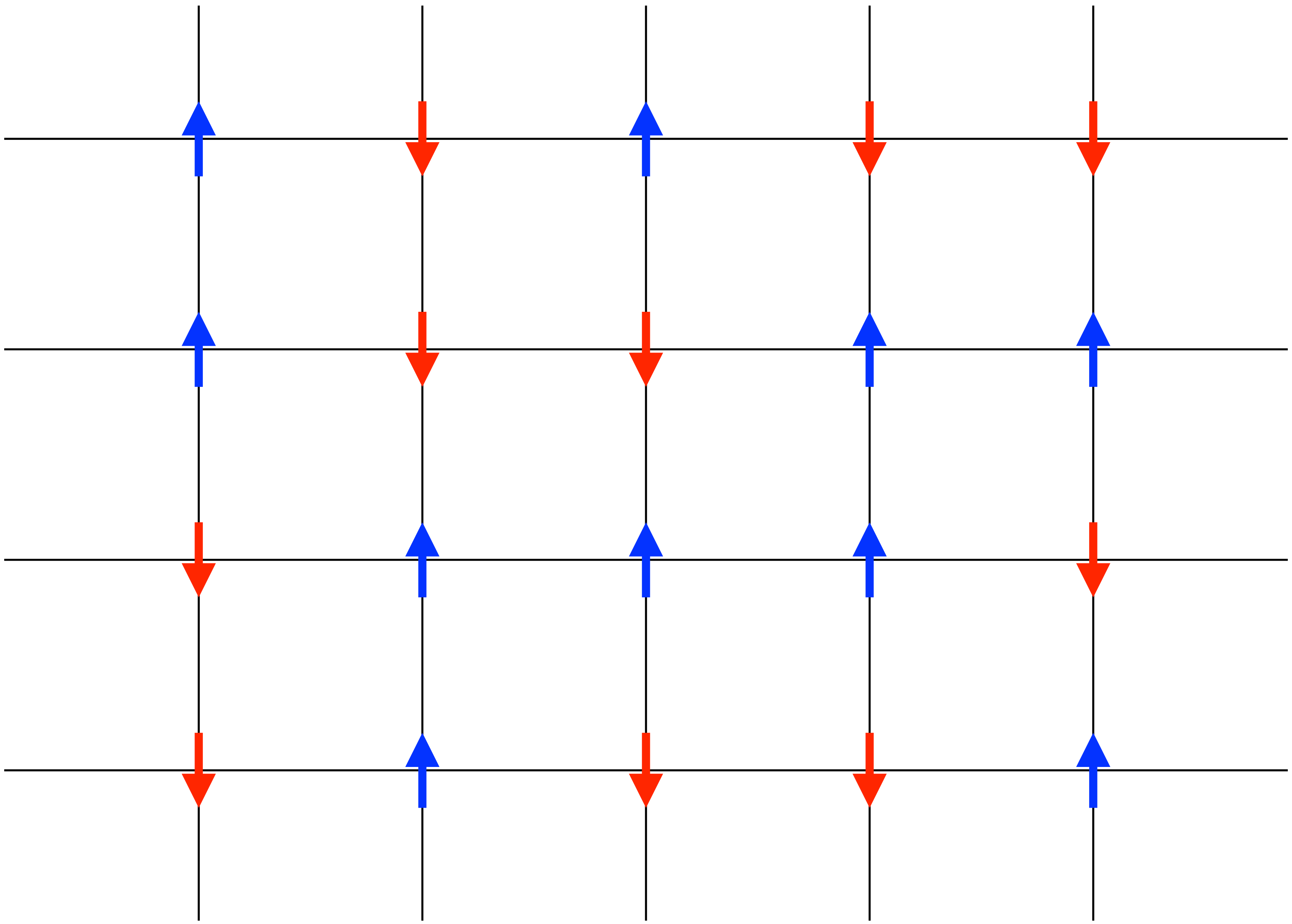
At high temperature T, the entropy term is dominant.

$\Rightarrow$ system maximizes entropy S

At low temperature T, the entropy term is negligible.

$\Rightarrow$ system minimizes energy E at the cost of S

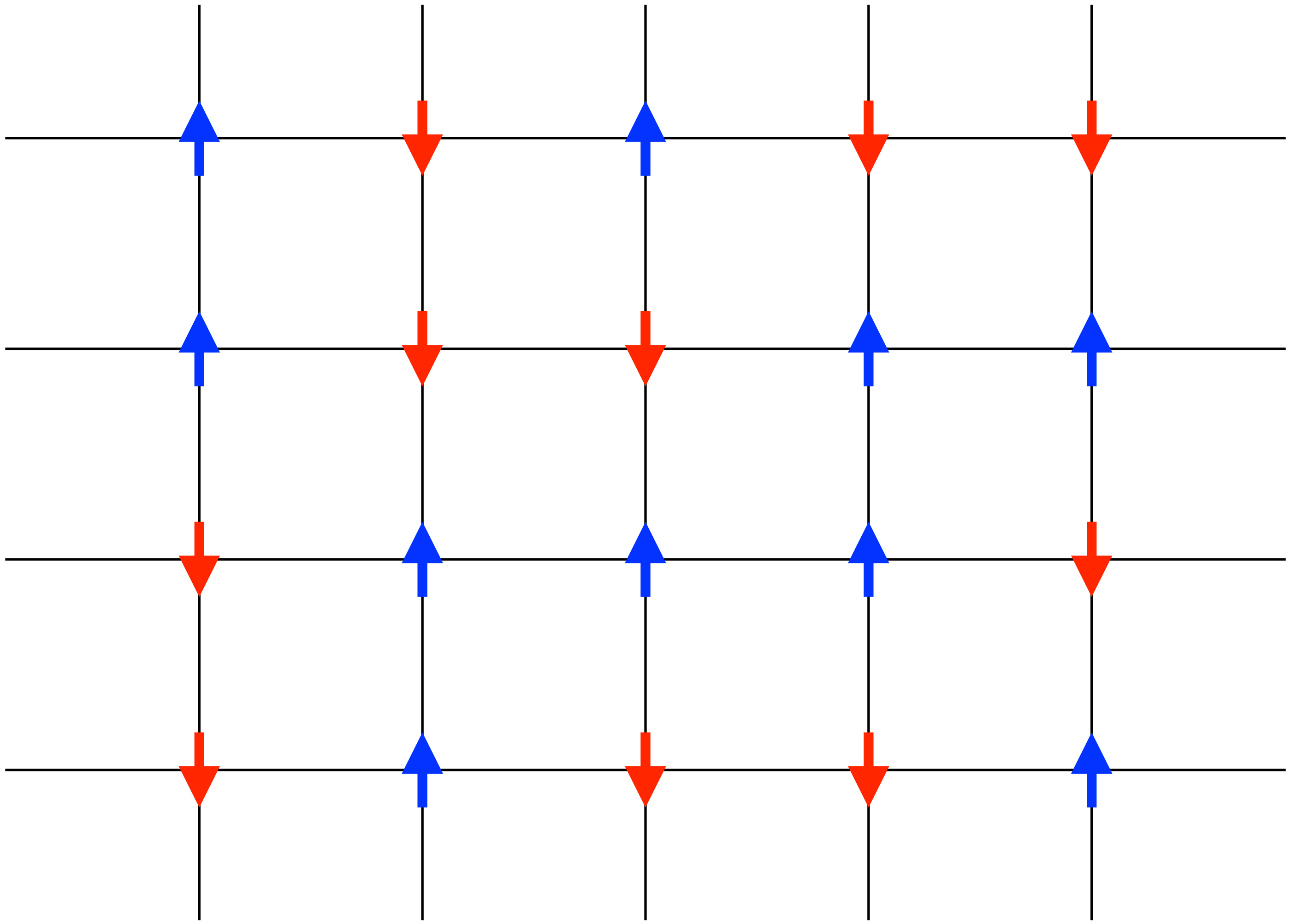
Example of SSB: ferromagnet



$$H = -\sum_{\langle i,j \rangle} s_i s_j$$

Example of SSB: ferromagnet

<u>**High T**</u>



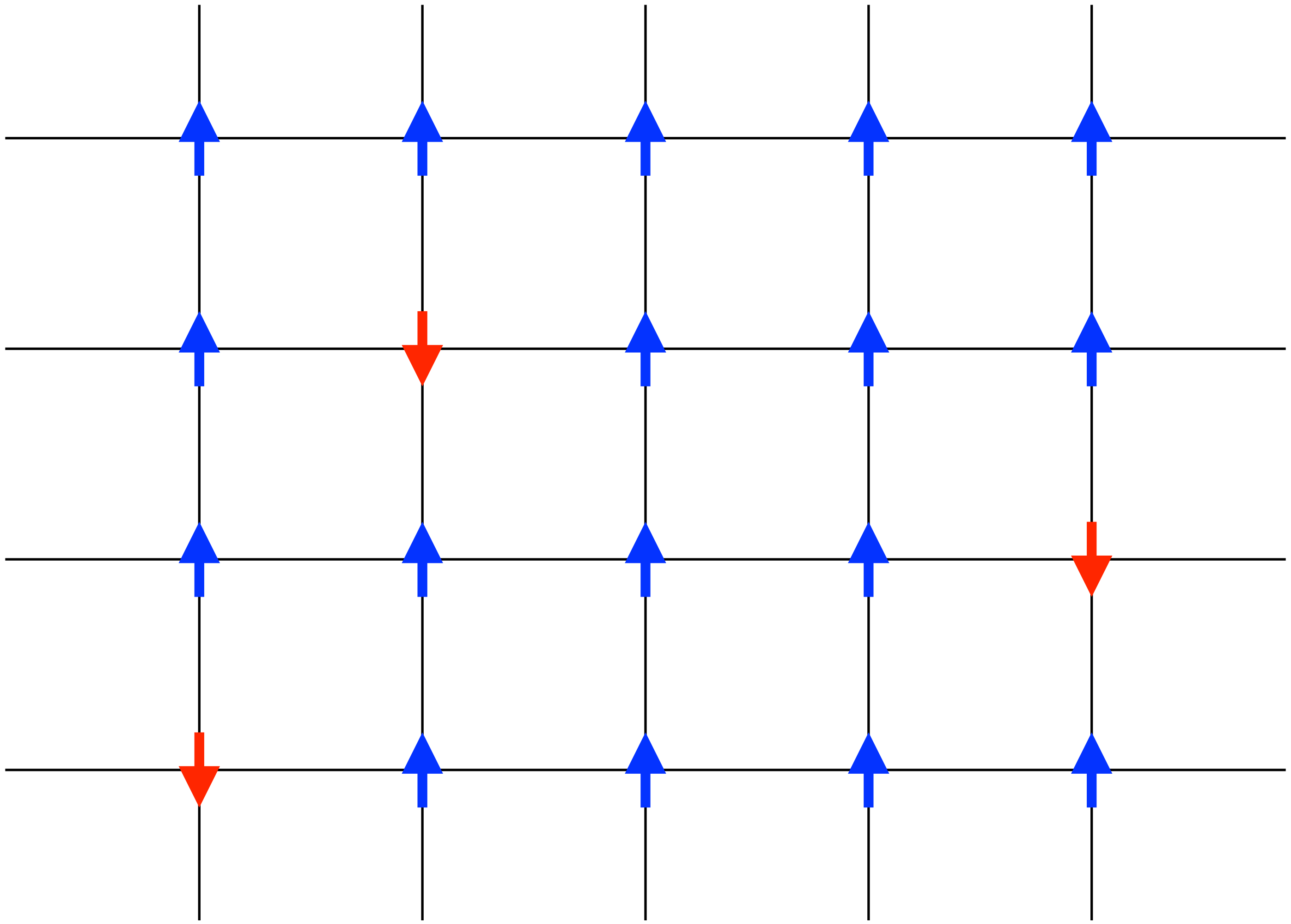
$$F = E - TS$$

$\Rightarrow$ To minimize F, random config. w/ large S is favored

$\Rightarrow$ Rotation symmetry is preserved

Example of SSB: ferromagnet

<u>Low T</u>



$$F = E - TS$$

⇒ To minimize F, aligned config. w/ small E is favored

⇒ SSB of rotation sym.

SSB = F minimization

Symmetry\Gap	Gapped (or TQFT)	Gapless (~CFT)
Preserved		
Spont. broken		

Another possibility: emergent symmetry

Sometimes, Sym.$_{UV}$ $\subset$ Sym.$_{IR}$.

ex)

- 90° rotation (square lattice) $\subset$ Lorentz (continuum)
- 4d $\mathcal{N}=1$ Lagrangian (UV) $\subset$ 4d $\mathcal{N}=2$ (IR)
- $SU(8)$ flavor $\subset$ E_7 flavor (4d $\mathcal{N}=1$ $SU(2)$ SQCD w/ $N_f=4$)

When, Why, What?

- When and why symmetry emerges?
- What is its structure (`size' and `algebra')?
- Can emergent sym. also be understood via F?

Q:**When** and **Why**

What symmetry emerges?

Content

1. Def. of Symmetry

2. (New) constraints on RG flow

3. Emergent Symmetry

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What are symmetries?

Traditional

$\exists$ charge Q which is

1. defined on time-slice,

2. and conserved.

What are symmetries?

Traditional

1. time-slice

$t = t_0$ —— $Q(\Sigma_{t=t_0})$

2. conserved

$t = t_0 + \delta t$ —— $Q(\Sigma_{t=t_0+\delta t})$

$||$

$t = t_0$ —— $Q(\Sigma_{t=t_0})$

What are symmetries?

Modern [Gaiotto-Kapustin-Seiberg-Willett '14]

$\exists$ charge Q which is

1. defined on time-slice,
   ||
   codimension-1 defect

2. and conserved.
   ||
   topological

Define symmetry by these two axioms.

Some generalizations

ordinary sym. := codimension-1 topological defect

- codim.-$(p+1)$ → p-form symmetry

- Non-invertible (=monoid) → non-invertible symmetry

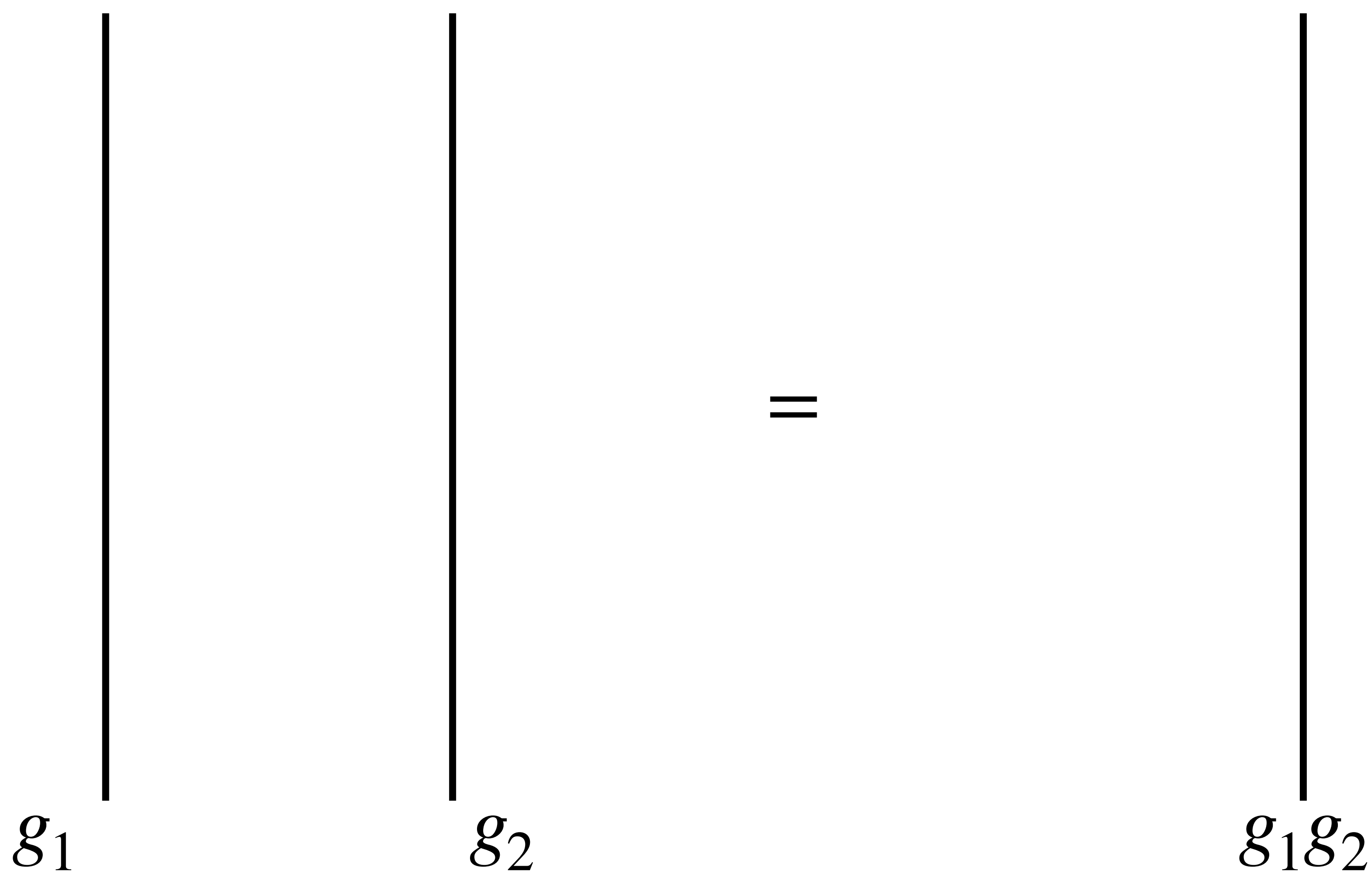
Some generalizations

ordinary sym. := codimension-1 topological defect

- codim.-$(p+1)$ → p-form symmetry
- Non-invertible (=monoid) → non-invertible symmetry
  or **category symmetry**

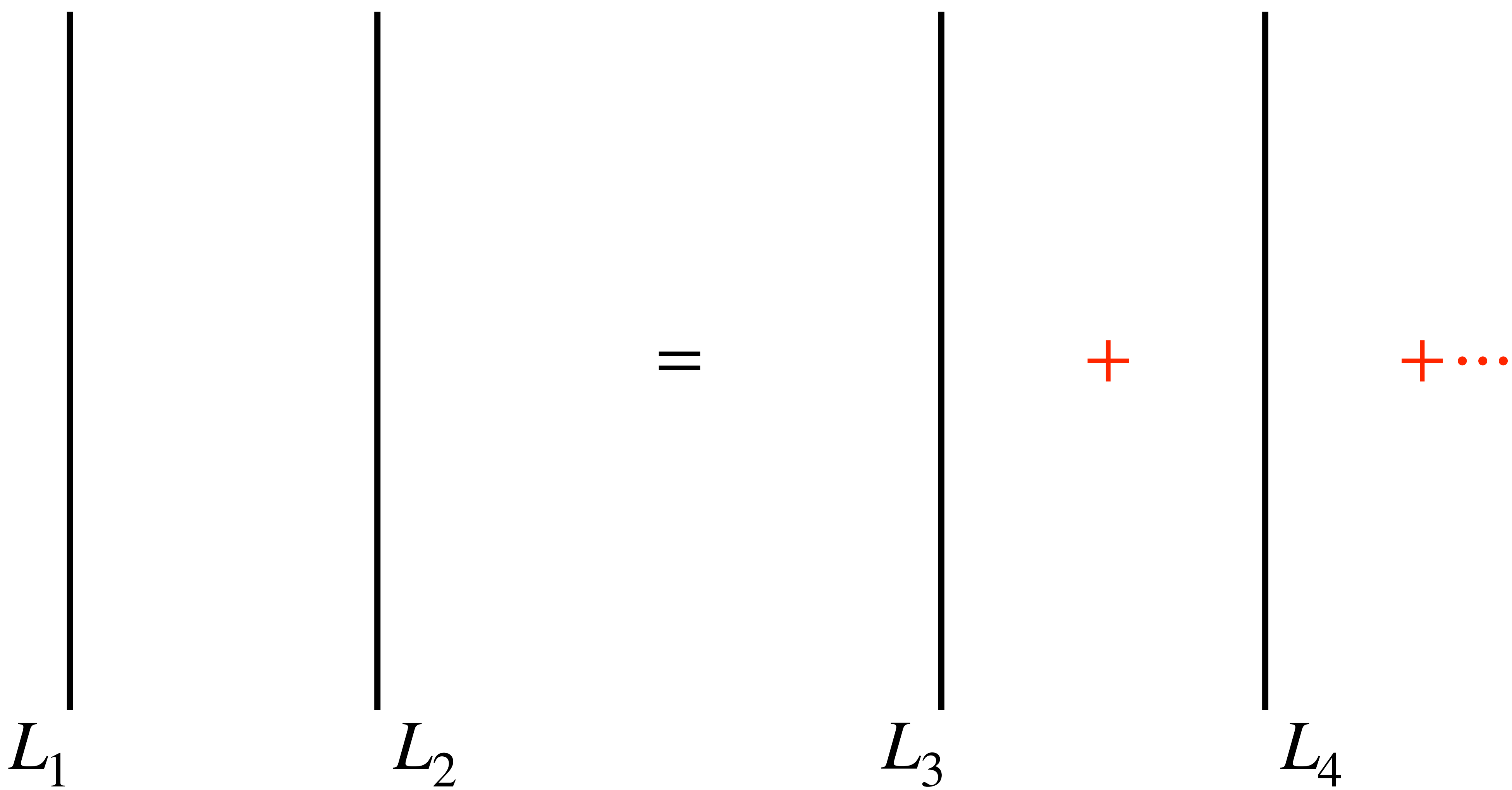
Category sym. vs. group

Group



Category sym. vs. group

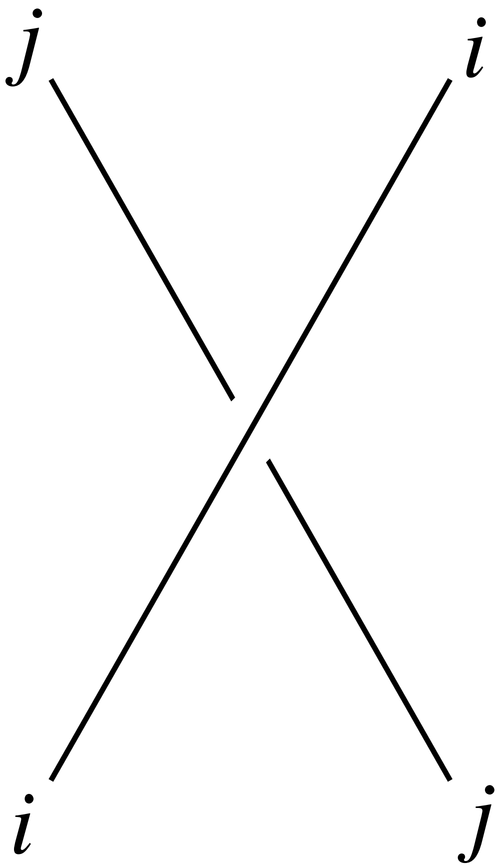
Category



Braided Fusion Category (BFC)

- Fusion category C = objects & fusion (w/ consistency)
- BFC (C, c) = fusion cat. w/ braiding c (w/ consistency)

$c_{i,j}$ =



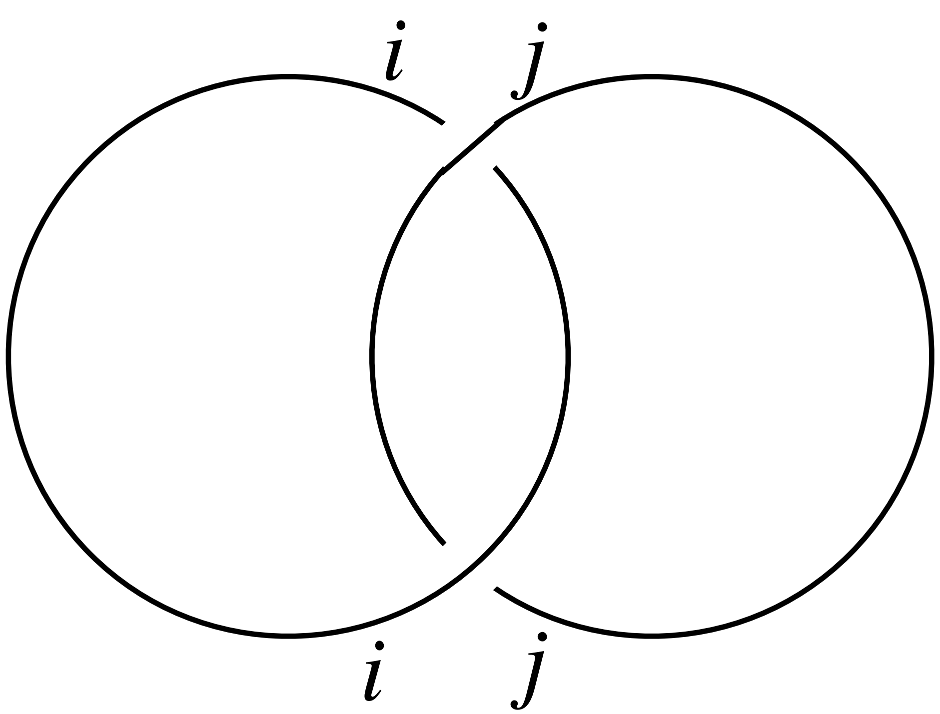
Modular Tensor Category (MTC)

- Fusion category C = objects & fusion (w/ consistency)

- BFC (C, c) = fusion cat. w/ braiding c (w/ consistency)

- MTC (C, c) = BFC w/ non-singular S-matrix (=modular)

$$\widetilde{S}_{ij} := \mathbf{tr}(c_{j,i}c_{i,j}) =$$



Example of category sym.

0-form sym. in 2d)

codim.-1 = line L

<u>$M(4,3)$ minimal model</u>

3 TDLs (topological defect lines) ~ charge

||

rank 3

$$L_{id}, \quad L_{\mathbb{Z}_2}, \quad L_N$$

Example of category sym.

2d $M(4,3)$ minimal model

`Algebra' of the TDLs:

$$L_{\mathbb{Z}_2} L_{\mathbb{Z}_2} = L_{id},$$

$$L_N L_{\mathbb{Z}_2} = L_N, \quad L_N L_N = L_{id} + L_{\mathbb{Z}_2}.$$

Example of category sym.

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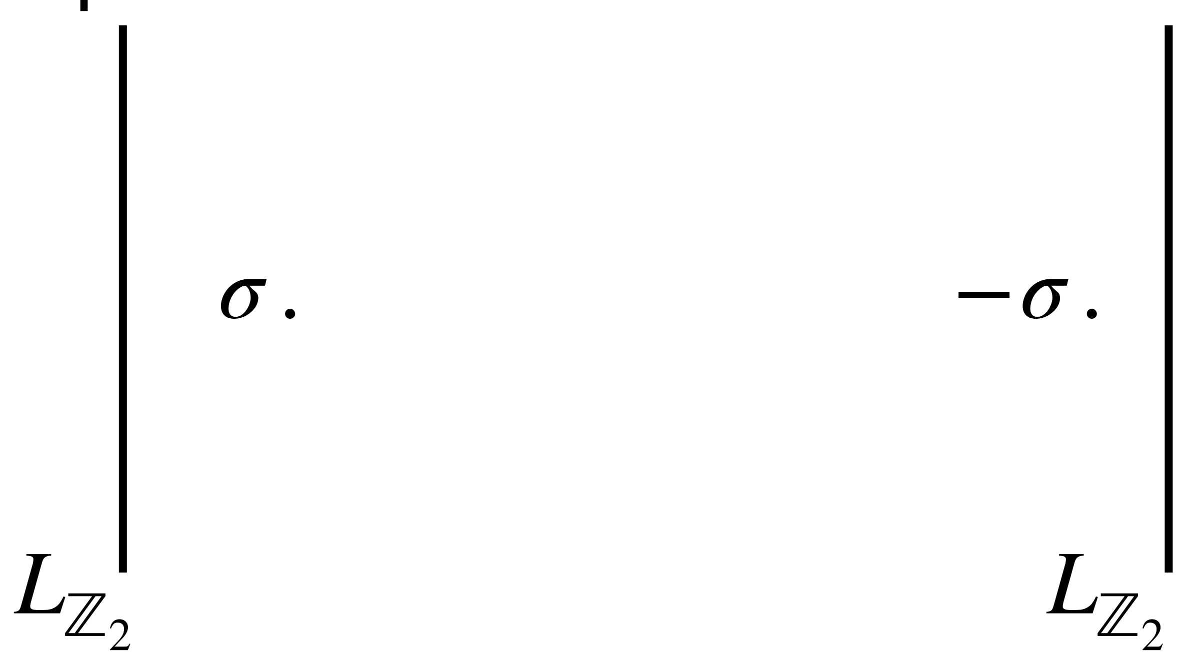
TDLs act on operators:



Example of category sym.

2d $M(4,3)$ minimal model

3 TDLs L_{id}, $L_{\mathbb{Z}_2}$, L_N have S-matrix

$$\widetilde{S} = \left(\begin{array}{cc:c} 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \\ \hdashline \sqrt{2} & -\sqrt{2} & 0 \end{array}\right).$$

$\{L_{id}, L_{\mathbb{Z}_2}, L_N\}$ is **modular**, while $\{L_{id}, L_{\mathbb{Z}_2}\}$ is **non-modular**.

Q:When and Why

What (category) sym. emerges?

A: Consistent category w/ minimal F.

Content

1. Def. of Symmetry
2. (New) constraints on RG flow
3. Emergent Symmetry

Constraints on RG flow

We will present 2 types of new constraints:

- “Monotonicity”
  - Spin constraint
  - Scaling dimension
  - Global dimension
- Double braiding relation

Constraints on RG flow

A TDL L is preserved along RG flow triggered by an op. $\mathcal{O}$ if it commutes with $\mathcal{O}$.

[Gaiotto '12, Chang-Lin-Shao-Wang-Yin '18]

New spin constraint

When relevant ops. are scalars, rotation sym. is preserved.

$\Rightarrow$ Spins are conserved.

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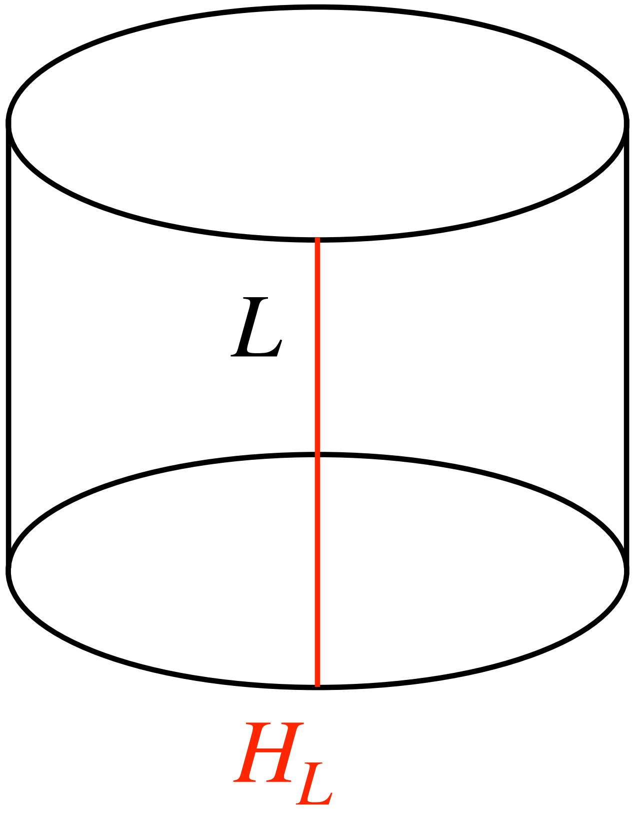
For a TDL L, we have an associated defect Hilbert space H_L.



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For a TDL L, we have an associated defect Hilbert space H_L.

Operators of H_L have specific spins, called spin contents S_L.

$\Rightarrow S_L$ of surviving TDL L are (basically) preserved.

$$S_L^{IR} \subset S_L^{UV}$$

[KK-Chen-Xu-Chang ’22]

Monotonic decrease of scaling dim.

In Wilsonian RG, we integrate out heavy modes.

$\Rightarrow$ EFTs should have lighter observables.

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In Wilsonian RG, we integrate out heavy modes.

$\Rightarrow$ EFTs should have lighter observables.

In fact, we proved the monotonic decrease

$$h_j^{IR} \leq h_j^{UV}$$

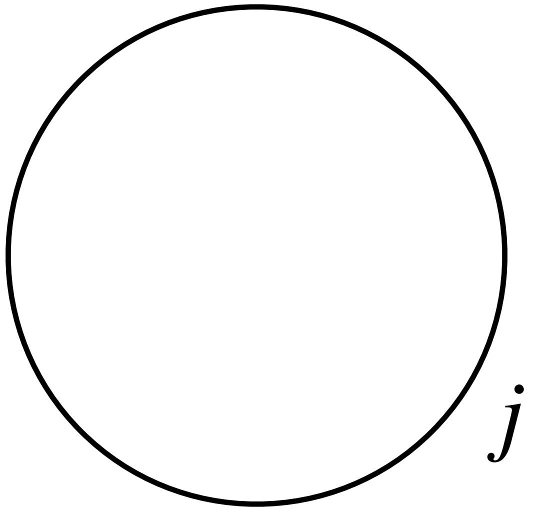
in minimal models (both unitary and non-unitary).

[2207.06433 (KK), 2207.10095 (KK)]

Monotonic decrease of global dim.

We can shrink away loops of TDLs.

$d_j :=$



Global dimension $\quad D^2 := \sum_j d_j^2$

Monotonic decrease of global dim.

A MTC w/ global dim. D gives entropy

$$S \ni -\ln D .$$

[Kitaev-Preskill '05, Levin-Wen '05]

The 2nd law of thermodynamics motivates

$$|D^{IR}| < |D^{UV}| .$$

In fact, we proved this in (non)unitary minimal models.

[2207.10095 (KK)]

Known monotonicity

c-theorem (unitary) **[Zamolodchikov ’86]**

$$(0 <)c_{IR} < c_{UV}$$

Effective c-theorem (non-unitary) **[Castro-Alvaredo-Doyon-Ravanini ’17]**

$$(0 \leq)c^{eff}_{IR} \leq c^{eff}_{UV}$$

$$c^{eff} := c - 24h_{smallest}$$

Summary of monotonicities

- (Effective) c-theorem $c_{IR} < c_{UV}$

  $c_{IR}^{eff} \leq c_{UV}^{eff}$

- Spin constraint $S_L^{IR} \subset S_L^{UV}$

- Scaling dimension $h_j^{IR} \leq h_j^{UV}$

- Global dimension $|D^{IR}| < |D^{UV}|$

Double braiding relation

"Something discrete cannot jump under RG flow."

e.g. 't Hooft anomaly matching

Double braiding relation

“Something discrete cannot jump under RG flow.”

e.g. ’t Hooft anomaly matching

Wrong!

Double braiding relation

Braiding is subject to consistency. (hexagon axiom)

Its solutions are discrete. (Ocneanu rigidity)

$\Rightarrow$ Braidings should **NOT** jump under RG flow, right?

Let's see a 'counterexample.'

Double braiding relation

$M(5,4)$ minimal model (UV)

↓

$M(4,3)$ minimal model (IR)

Double braiding relation

3 TDLs $\{L_{id}, L_{\mathbb{Z}_2}, L_N\}$ of $M(5,4)$ survive the flow.

L_N line has double braiding

$$c^{UV}_{L_N,L_N} c^{UV}_{L_N,L_N} = e^{\pi i/4} id_{L_{id}} \oplus e^{5\pi i/4} id_{L_{\mathbb{Z}_2}}.$$

But $M(4,3)$ has

$$c^{IR}_{L_N,L_N} c^{IR}_{L_N,L_N} = e^{-\pi i/4} id_{L_{id}} \oplus e^{-5\pi i/4} id_{L_{\mathbb{Z}_2}}.$$

$\Rightarrow$ Braidings jump!

Double braiding relation

You may realize double braidings $c_{j,i}c_{i,j}$ are the opposite

$$c^{IR}_{j,i}c^{IR}_{i,j} = (c^{UV}_{j,i}c^{UV}_{i,j})^* \,.$$

This relation holds in all examples we studied.

Why?

RG interface

[Gaiotto '12]

RG flow = maps : UV ops. → IR ops.

UV RCFT

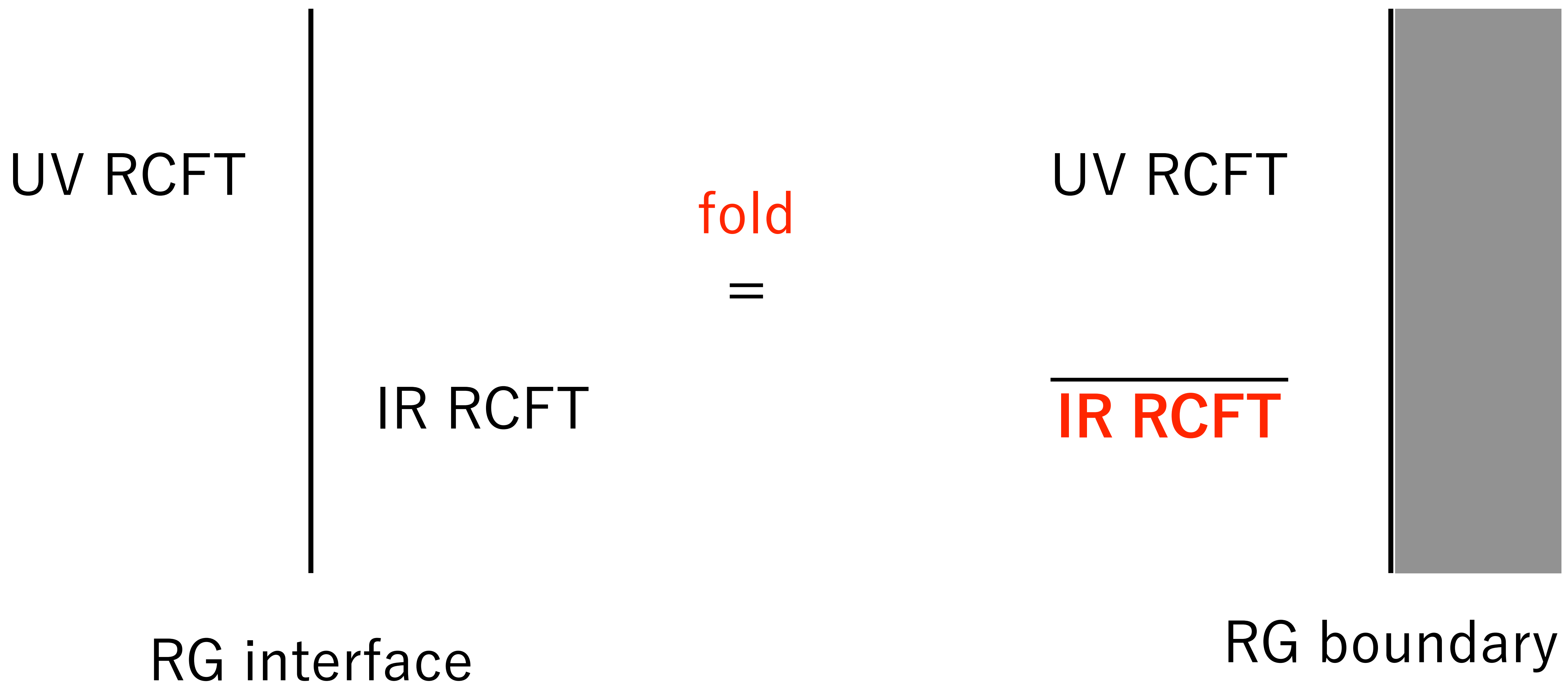
IR RCFT

RG interface

RG interface

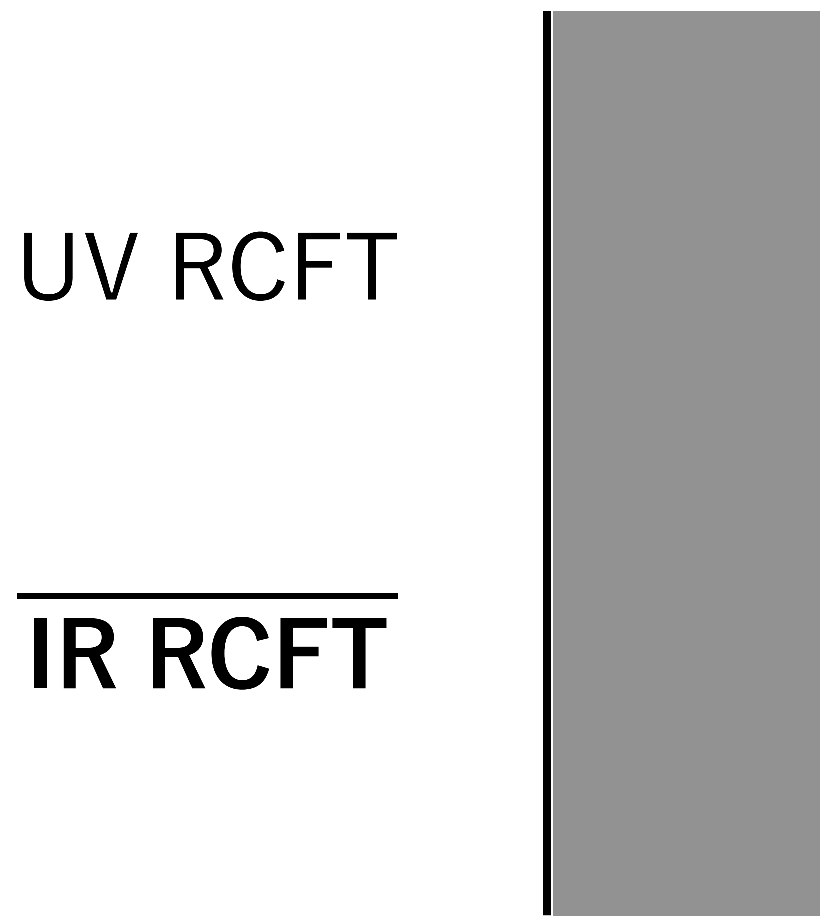
[Gaiotto '12]

RG flow = maps : UV ops. → IR ops.



RG interface

[Gaiotto '12]

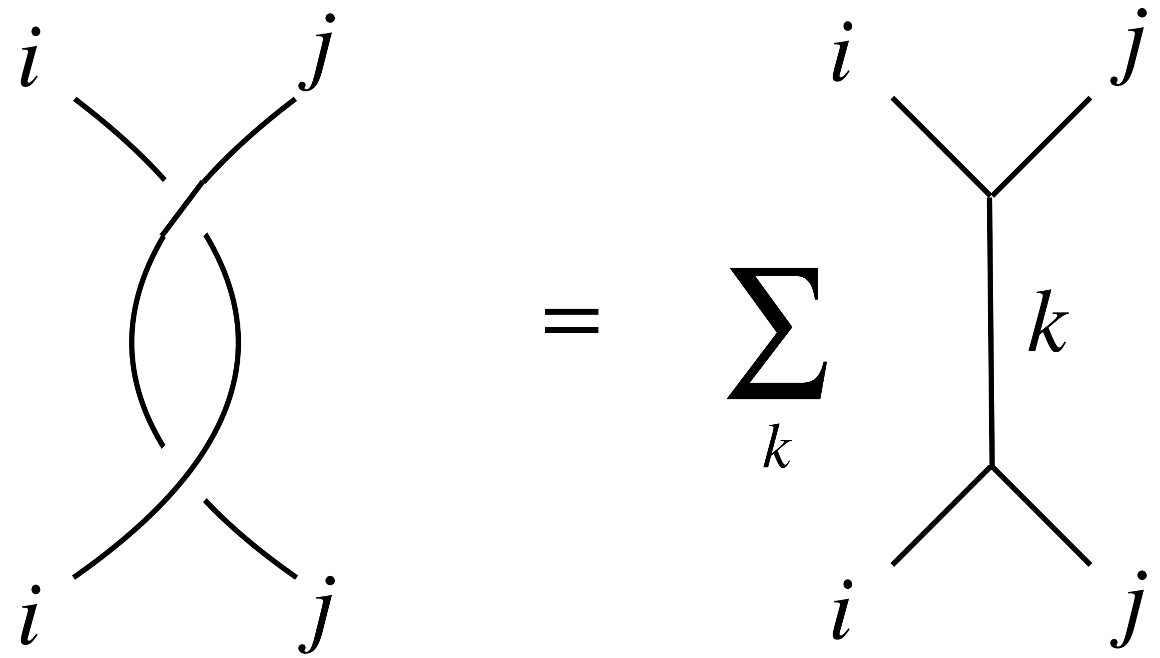


The folding trick turns
right-handed rule (UV) to left-handed rule (IR).

$$\Rightarrow c^{IR}_{j,i} c^{IR}_{i,j} = (c^{UV}_{j,i} c^{UV}_{i,j})^*$$

Prediction of conf. dim. h_j

Double braiding is given by



$$c_{j,i}c_{i,j} = \sum_k N_{ij}^k \frac{e^{2\pi i h_k}}{e^{2\pi i h_i} e^{2\pi i h_j}} id_k \,.$$

⇒Prediction on IR h_j!

Sample prediction of h_j

$$c_{j,i}c_{i,j} = \sum_k N_{ij}^k \frac{e^{2\pi i h_k}}{e^{2\pi i h_i} e^{2\pi i h_j}} id_k \,.$$

$M(5,4)$ model has

$$c_{L_N,L_N} c_{L_N,L_N} = e^{\pi i/4} id_{L_{id}} \oplus e^{5\pi i/4} id_{L_{\mathbb{Z}_2}} \,.$$

The L_{id}-channel predicts

$$e^{-4\pi i h_{L_N}^{IR}} = e^{-\pi i/4}$$

or

$$h_{L_N}^{IR} = \frac{1}{16} \text{ mod } \frac{1}{2}.$$

Sample prediction of h_j

The smallest positive candidate $1/16$ is physically favored

because h enters energy $\hat{L}_0 | h \rangle = h | h \rangle$.

$$\Rightarrow \text{Correct prediction } h^{IR}_{L_N} = \frac{1}{16}.$$

We also get

$$e^{2\pi i h^{IR}_{L_{\mathbb{Z}_2}}} = e^{(1+3)\pi i/4}$$

or

$$h^{IR}_{L_{\mathbb{Z}_2}} = \frac{1}{2}.$$

Comment on gap

The double braiding relation can rule out gapless scenario.

Pick 3 RCFTs T_1, T_2, T_3 w/ RG flow

$$T_1 \to T_2 \to T_3 \,.$$

The double braiding relation claims

$$c_{j,i}^{T_1} c_{i,j}^{T_1} = (c_{j,i}^{T_2} c_{i,j}^{T_2})^* = c_{j,i}^{T_3} c_{i,j}^{T_3} = (c_{j,i}^{T_3} c_{i,j}^{T_3})^* \,.$$

$\Rightarrow$ Such pairs i, j should have real double braiding.

Comment on gap

The reality condition $c_{j,i}c_{i,j} = (c_{j,i}c_{i,j})^*$ explains why

- $M(5,4)$ minimal model+σ'
- $M(6,5)$ minimal model+ϵ'

are gapped.

It also explains some structures of theory space.

Content

1. Def. of Symmetry
2. (New) constraints on RG flow
3. Emergent Symmetry

Specialize to 2d RCFT

Facts

- Sym. (sub)category = MTC (modular tensor category)

  =BFC w/ non-singular S-matrix

- c-theorem: $c_{IR} < c_{UV}$

Specialize to 2d RCFT

Facts

- Sym. (sub)category = MTC (modular tensor category)
- c-theorem: $c_{IR} < c_{UV}$

$\Rightarrow$ If IR theory is RCFT,
its sym. category should be MTC w/ $c < c_{UV}$.

[KK '21]

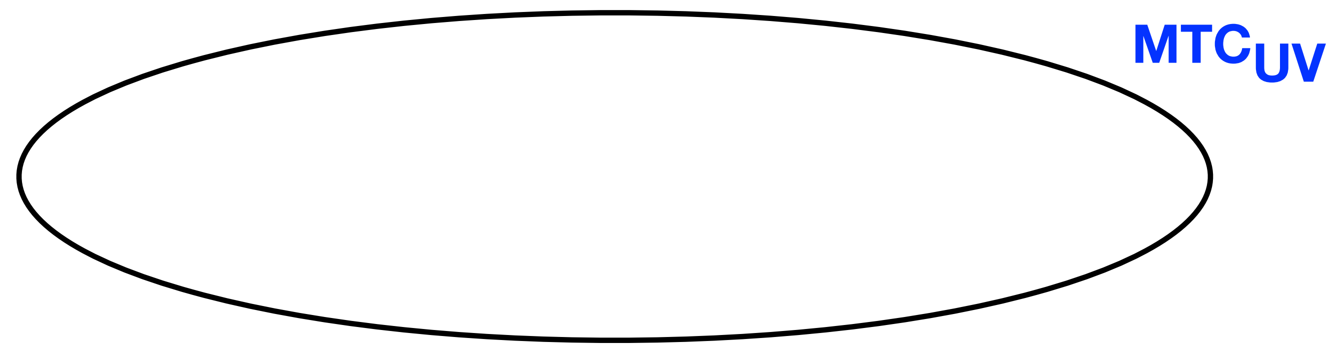
Our example

$M(m+1,m)$ minimal model

$\Big\downarrow \; +\phi_{1,3}$

$M(m,m-1)$ minimal model

Key idea



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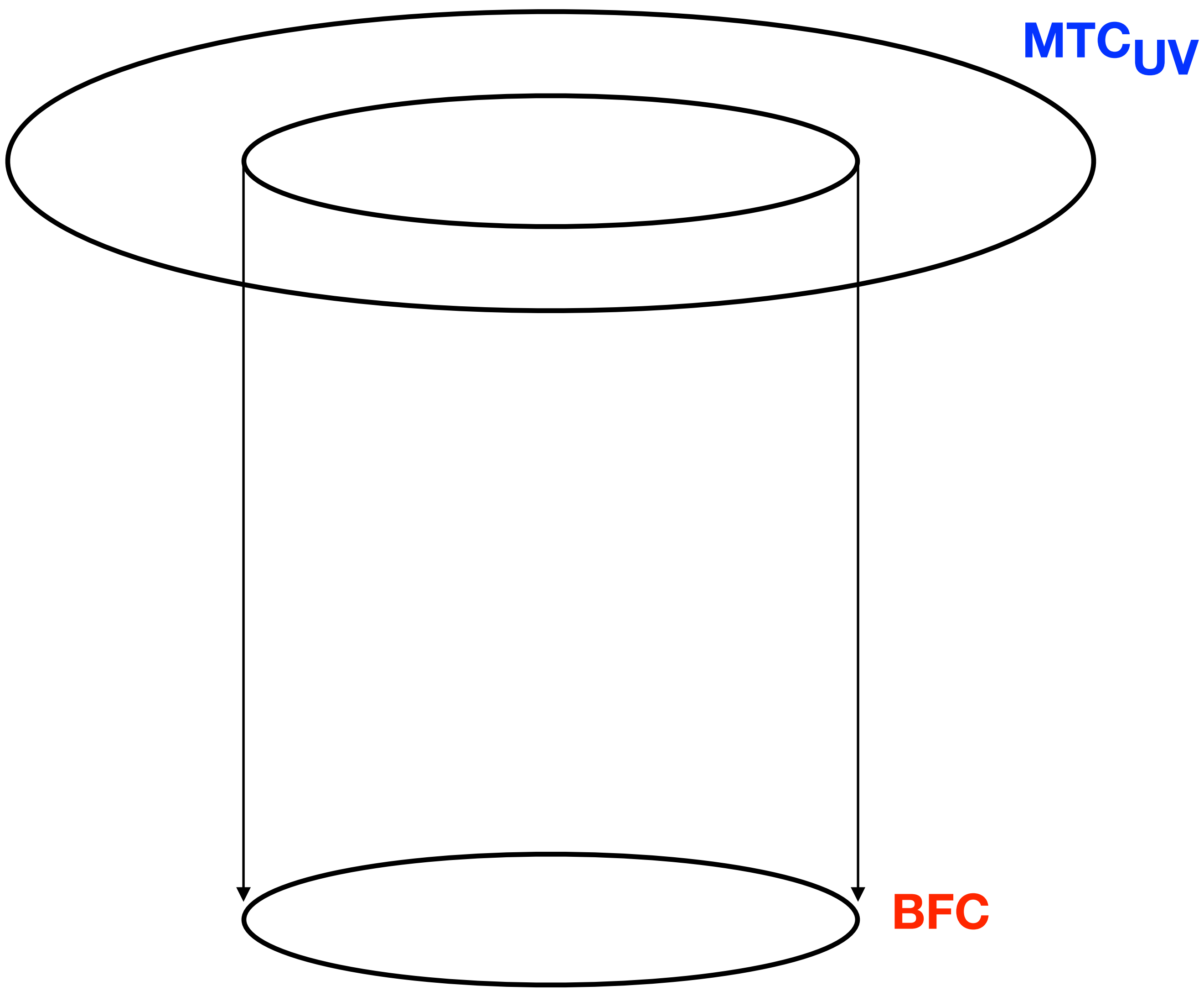

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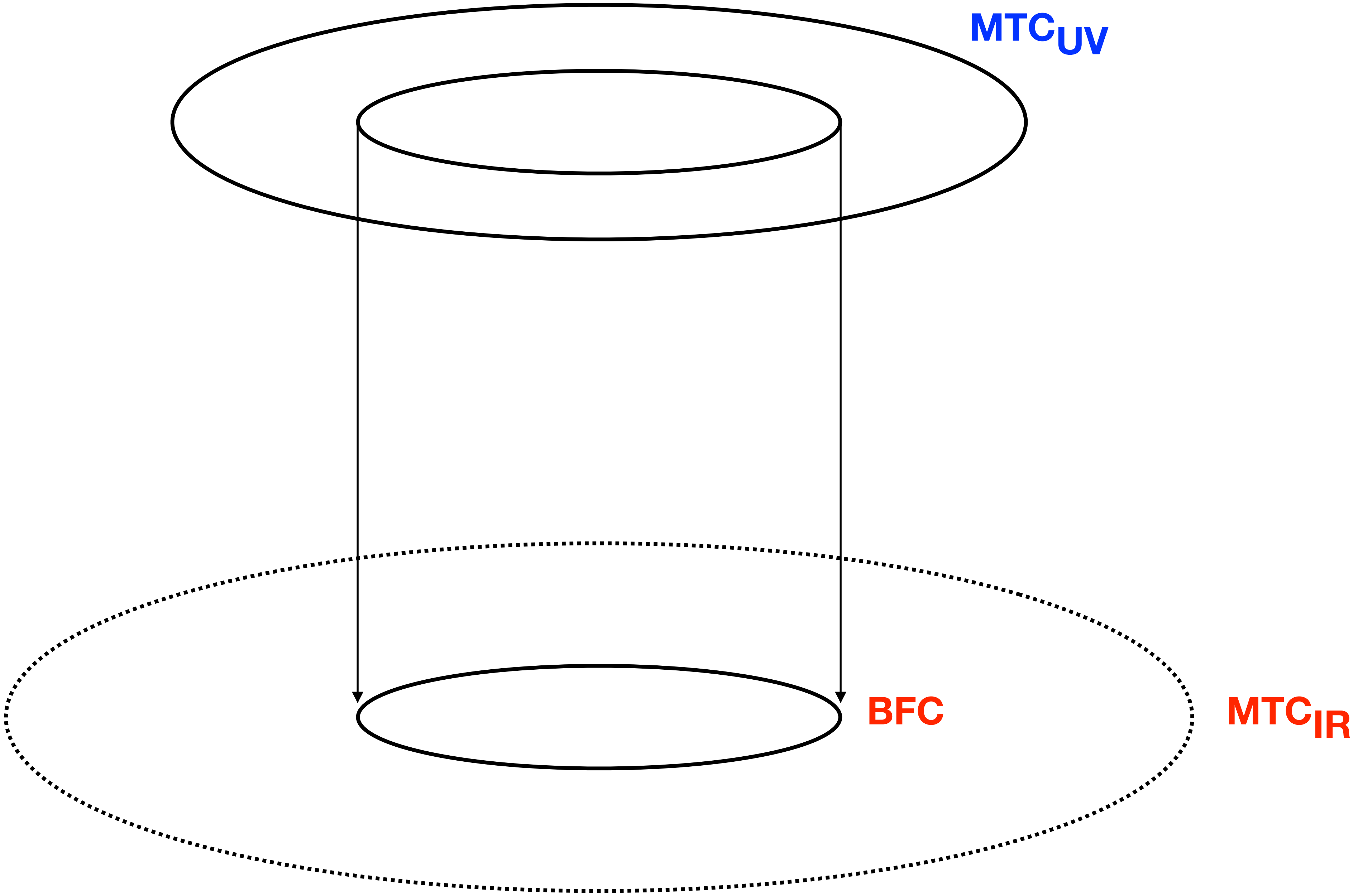

Test1) $M(5,4)$ minimal model$+\phi_{1,3}$

- The theory has 6 TDLs.

1. Only 3 of them $\{L_{id}\ , L_{2,1}\ , L_{3,1}\}$ survive $\phi_{1,3}$-deform.

2. They can form rank 3 MTC w/ $c < c_{UV} = \frac{7}{10}$.

[Gepner-Kapustin '94]

3. Emergent sym. is unnecessary.

Test1) $M(5,4)$ minimal model$+\phi_{1,3}$

Absence of emergent sym. is consistent with known RG flow

$M(5,4)$ minimal model (UV)

$\downarrow$ $+\phi_{1,3}$

$M(4,3)$ minimal model (IR)

Test2) $M(6,5)$ minimal model+$\phi_{1,3}$

- The theory has 10 TDLs.

1. Only 4 $\{L_{id}, L_{2,1}, L_{3,1}, L_{4,1}\}$ survive $\phi_{1,3}$-deform.

2. The rank 4 sym. category C is not an MTC.

3. Emergent sym. is needed!

Test2) $M(6,5)$ minimal model+$\phi_{1,3}$

Can 1 emergent TDL make C an MTC?

Test2) $M(6,5)$ minimal model+$\phi_{1,3}$

Can 1 emergent TDL make C an MTC?

No. There is no rank 5 MTC containing C.

[Bruillard-Ng-Rowell-Wang '15]

Test2) $M(6,5)$ minimal model+$\phi_{1,3}$

How about 2 emergent TDLs?

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How about 2 emergent TDLs?

It works.

There is one rank 6 MTC w/ $c < c_{UV} = \frac{4}{5}$.

[Gepner-Kapustin '94]

Test2) $M(6,5)$ minimal model $+\phi_{1,3}$

The emergent sym. is consistent with known RG flow

$M(6,5)$ minimal model (UV)

$\Big\downarrow +\phi_{1,3}$

$M(5,4)$ minimal model (IR)

Remark on free energy

Consistent MTC w/ minimal rank is realized.

In unitary theory, $d_j \geq 1$, and larger rank has larger D.

$\Rightarrow$ Minimal free energy $\simeq$ Minimal rank

Test3) $M(7,6)$ minimal model$+\phi_{1,3}$

- The theory has 15 TDLs.

1. Only 5 $\{L_{id}, L_{2,1}, L_{3,1}, L_{4,1}, L_{5,1}\}$ survive.
2. The rank 5 sym. category C is an MTC. [Gepner-Kapustin '94]

But how about c?

Test3) $M(7,6)$ minimal model+$\phi_{1,3}$

- According to [Gepner-Kapustin '94], the rank 5 MTC has

$$c = 2 \text{ mod } 4.$$

- Assuming unitarity, this means

$$c = 2,6,10,\ldots.$$

- This cannot be smaller than $c_{UV} = \frac{6}{7}$!

- Symmetry should emerge.

Test3) $M(7,6)$ minimal model+$\phi_{1,3}$

Can 1 emergent TDL make C consistent with c-theorem?

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Unfortunately, rank $r \geq 7$ MTCs are poorly classified…

Test3) $M(7,6)$ minimal model+$\phi_{1,3}$

How about 2 emergent TDLs?

Unfortunately, rank $r \geq 7$ MTCs are poorly classified⋯

At least, no candidate up to $r = 9$ in the partial classification [Wen '15], suggesting $r \geq 10$.

Test3) $M(7,6)$ minimal model$+\phi_{1,3}$

However, ∃emergent sym. itself is

consistent with known RG flow

$M(7,6)$ minimal model (UV)

$$\Big\downarrow +\phi_{1,3}$$

$M(6,5)$ minimal model (IR)

Symmetry enhancement in RCFT

2 requirements

・modularity
・c-theorem

correctly explain sym. enhancement in 2d unitary RCFT.

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・modularity
・c-theorem

correctly explain sym. enhancement in 2d unitary RCFT.

How about non-unitary cases?

⇒Use c^{eff}-theorem.

Non-unitary example

$M(p, 2p+1)$ minimal model

$\downarrow$ $+\phi_{5,1}$

$M(p, 2p-1)$ minimal model

$\downarrow$ $+\phi_{1,2}$

$M(p-1, 2p-1)$ minimal model

Test4) $M(3,5)$ minimal model+$\phi_{1,2}$

- The theory has 4 TDLs.

1. Only 2 $\{L_{id}, L_{3,1}\}$ survive.
2. They can form rank 2 MTC. [Gepner-Kapustin '94]

Consistent w/ c^{eff}-theorem?

$\Rightarrow$Compute c^{eff} by predicting h^{IR}

Test4) $M(3,5)$ minimal model+$\phi_{1,2}$

$L_{3,1}$ has

$$c_{L_{3,1},L_{3,1}}c_{L_{3,1},L_{3,1}} = e^{-4\pi i/5} id_1 \oplus e^{-2\pi i/5} id_{3,1}\,.$$

Thus, $L_{3,1} \to j$ should have

$$c_{j,j}c_{j,j} = e^{+4\pi i/5} id_1 \oplus e^{+2\pi i/5} id_j\,.$$

The j-channel predicts

$$e^{-2\pi i h_j^{IR}} = e^{2\pi i/5},$$

or

$$h_j^{IR} = \frac{4}{5}\ (\text{mod } 1).$$

Test4) $M(3,5)$ minimal model$+\phi_{1,2}$

With "monotonicity" $h_j^{IR} \leq \frac{1}{5}$, we get

$$h_j^{IR} \leq -\frac{1}{5}.$$

c^{eff}-theorem $c_{IR} - 24h_j^{IR} \leq c_{UV}^{eff} = \frac{3}{5}$ gives

$$c_{IR} \leq -\frac{21}{5}.$$

The rank 2 MTC can have such c_{IR}. [Gepner-Kapustin '94]

$\Rightarrow$ Emergent symmetry is unnecessary.

Test4) $M(3,5)$ minimal model+$\phi_{1,2}$

Absence of emergent sym. is consistent with known RG flow

$M(3,5)$ minimal model (UV)

$\Big\downarrow +\phi_{1,2}$

$M(2,5)$ minimal model (IR)

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

- The theory has 6 TDLs.

1. Only 2 $\{L_{id}, L_{1,2}\}$ survive.

2. They can form rank 2 MTC C. [Gepner-Kapustin '94]

Consistent w/ c^{eff}-theorem?

$\Rightarrow$Compute c^{eff} by predicting h^{IR}

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

Double braiding relation predicts

$$h_j^{IR} = \frac{1}{4}\ (\mathrm{mod}\ \frac{1}{2}).$$

Small computation shows $c^{eff} \not\leq c_{UV}^{eff}$.

⇒Symmetry should emerge.

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

Can 1 emergent TDL make C consistent?

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

Can 1 emergent TDL make C consistent?

There is 1 rank 3 MTC enlarging C, but … [Gepner-Kapustin '94]

The scenario requires $d_L = 0$, contradiction.

(invertibility of F-symbols)

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

How about 2 emergent TDLs?

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

How about 2 emergent TDLs?

There are 3 rank 4 MTCs enlarging C: [Gepner-Kapustin '94]

Rank 4 MTCs	Possible?
$SU(4)_1$	
$SO(8)_1$	
$SU(2)_3$	

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

How about 2 emergent TDLs?

There are 3 rank 4 MTCs enlarging C: [Gepner-Kapustin '94]

Rank 4 MTCs	Possible?
$SU(4)_1$	X
$SO(8)_1$	
$SU(2)_3$	

(reality of d_j)

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

Which is realized, $SO(8)_1$ or $SU(2)_3$?

Test5) $M(3,7)$ minimal model+$\phi_{5,1}$

Which is realized, $SO(8)_1$ or $SU(2)_3$?

Each scenario has global dimension

$$D^2_{SO(8)_1} = 4,$$

$$D^2_{SU(2)_3} = 5 \pm \sqrt{5}\,.$$

$SU(2)_3$ scenario w/ $D^2 = 5 - \sqrt{5}$ is preferred

because it has minimal free energy $F \ni T \ln D$.

Test5) $M(3,7)$ minimal model$+\phi_{5,1}$

$SU(2)_3$ can have $c^{eff} \leq c^{eff}_{UV} = \frac{5}{7}$, and indeed it is correct:

$M(3,7)$ minimal model (UV)

$\downarrow$ $+\phi_{5,1}$

$M(3,5)$ minimal model (IR)

General remark

- Consistency explained when symmetry emerges.
- Minimization of F further fixed what sym. is realized.

Test6) $M(4,7)$ minimal model$+\phi_{1,2}$

- The theory has 9 TDLs.

1. Only 3 $\{L_{id}\ , L_{3,1}\ , L_{5,1}\}$ survive.

2. They can form rank 3 MTC C. [Gepner-Kapustin '94]

Consistent w/ c^{eff}-theorem?

⇒Compute c^{eff} by predicting h^{IR}

Test6) $M(4,7)$ minimal model+$\phi_{1,2}$

The rank 3 MTC C has $c = \frac{24}{7} - 4n$ w/$n \in \mathbb{N}$.

Double braiding relation predicts $L_{3,1} \to k, L_{5,1} \to j$ have

$$h_j = \frac{4}{7} - l,\ h_k = -\frac{1}{7} - m\ (l, m \in \mathbb{N}).$$

$$\begin{aligned}\therefore c^{eff} &= \left(\frac{24}{7} - 4n\right) - 24\min\left(\frac{4}{7} - l, -\frac{1}{7} - m\right) \\ &= \frac{4}{7}\left\{(6-7n) - \min(24 - 42l, -6 - 42m)\right\}\end{aligned}.$$

Test6) $M(4,7)$ minimal model$+\phi_{1,2}$

c^{eff}-theorem $0 \le c^{eff} \le c^{eff}_{UV} = \dfrac{11}{14}$ only allows

$$c^{eff} = \frac{4}{7}\left\{(6-7n) - \min(24-42l,\,-6-42m)\right\} = 0, \frac{4}{7},$$

or $b := \{\,\} = 0,1$. They have no solution:

$$18 + b = 7(-n + 6l),$$
$$-12 + b = 7(-n + 6m)\,.$$

⇒Symmetry should emerge (violate c^{eff}-theorem).

Test6) $M(4,7)$ minimal model$+\phi_{1,2}$

There is no rank 4,5 MTCs enlarging C.

[Gepner-Kapustin '94]

There are 2 rank 6 MTCs enlarging C:

Rank 6 MTCs	D^2
$SU(2)_5$	$2\times\frac{7}{4\cos^2\frac{\pi}{14}}$
$SU(2)_3/\mathbb{Z}_2\times SU(2)_5/\mathbb{Z}_2$	$\frac{5\pm\sqrt{5}}{2}\times\frac{7}{4\cos^2\frac{\pi}{14}}$

Test6) $M(4,7)$ minimal model+$\phi_{1,2}$

The 2nd w/$D^2 = \frac{5-\sqrt{5}}{2} \times \frac{7}{4\cos^2\frac{\pi}{14}}$ has minimal F.

But its c^{eff} is inconsistent w/ c^{eff}-theorem $\Rightarrow$ $SU(2)_5$.

Rank 6 MTCs	D^2
$SU(2)_5$	$2 \times \frac{7}{4\cos^2\frac{\pi}{14}}$
$SU(2)_3/\mathbb{Z}_2 \times SU(2)_5/\mathbb{Z}_2$	$\frac{5 \pm \sqrt{5}}{2} \times \frac{7}{4\cos^2\frac{\pi}{14}}$

Test6) $M(4,7)$ minimal model+$\phi_{1,2}$

Consistent $SU(2)_5$ w/ minimal F is indeed correct:

$M(4,7)$ minimal model (UV)

$\Big\downarrow +\phi_{1,2}$

$M(3,7)$ minimal model (IR)

Emergent symmetry

- Consistency explains when symmetries emerge.
- Min. free energy also explains what sym. is realized.
  - Rank,
  - Multiplication rule,
  - Anomaly, etc
- Agree with known RG flows in all examples.

Summary

Qualitative

- IR symmetry is realized by

  consistent symmetry category with minimal free energy.

- Discrete quantity can jump at conformal fixed point.

Summary

Quantitative

- We proposed mechanism behind emergent symmetry:

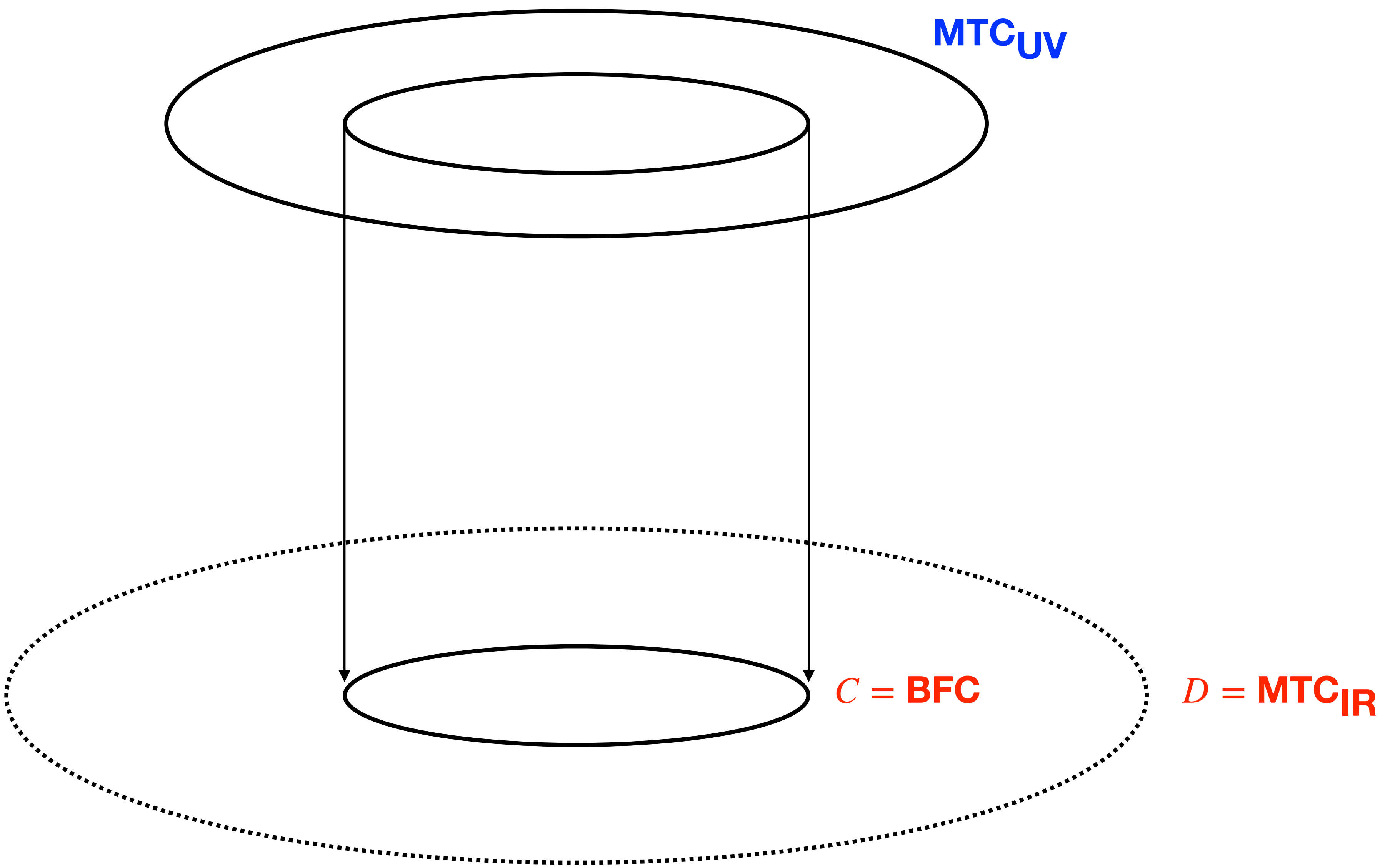
when surviving sym. category C is inconsistent

  - non-modularity, or
  - "non-monotonic,"

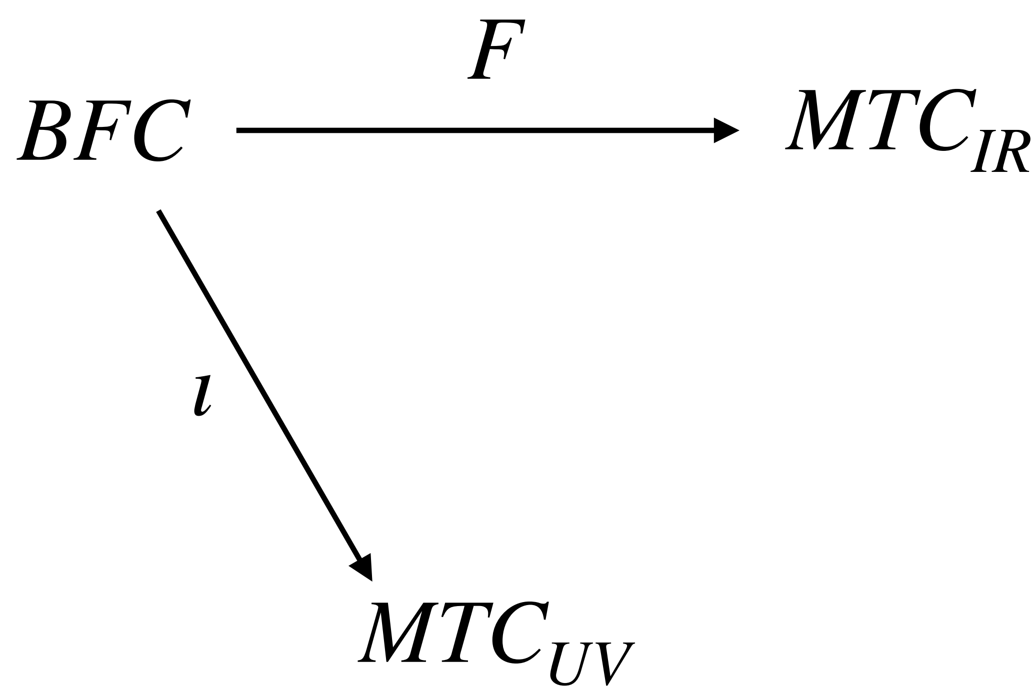
sym. should enhance to consistent $D \supset C$(=why) [KK '21]

w/ minimal free energy. [KK '22]

Summary



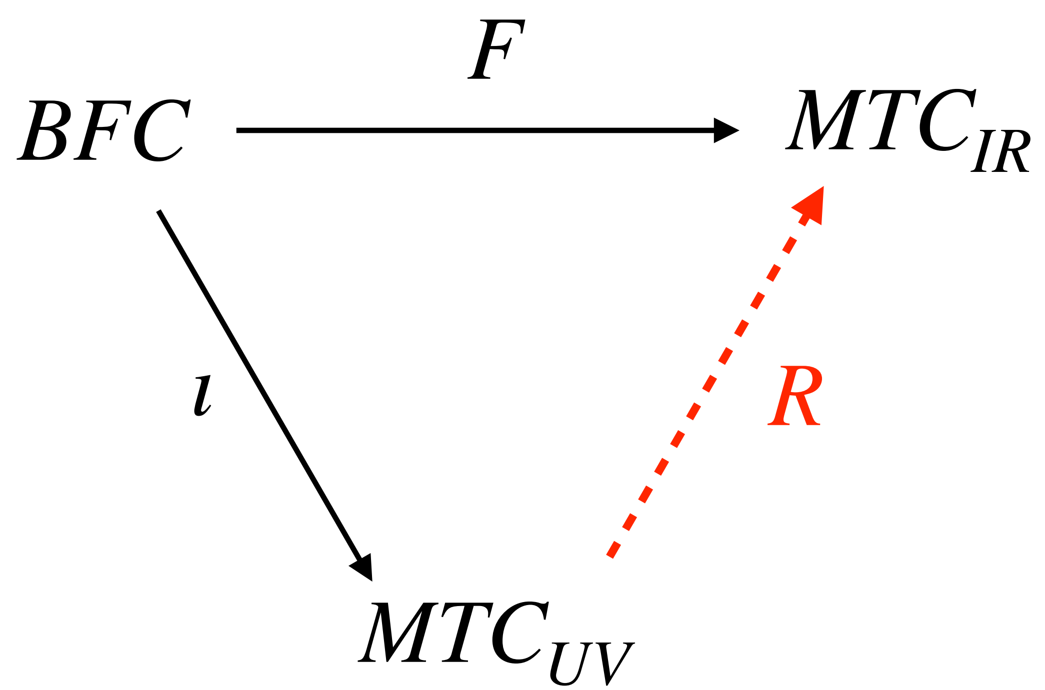
Summary



Summary

[2209.00016 (KK)]

- RG flows among RCFTs = Kan extension



Many future directions

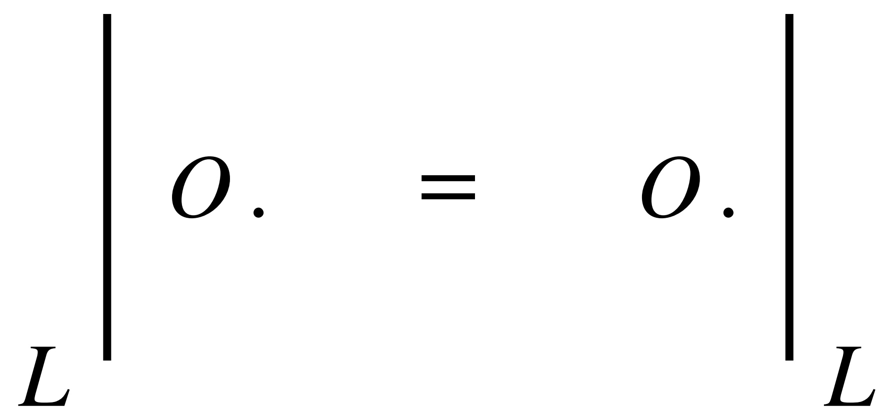
- More general RCFTs
  - Non-diagonal RCFT
  - Fermionic RCFT [KK-Chen-Xu-Chang '22] [2207.06433 (KK)] (commutativity)
  - Irrational CFT → RCFT
- RG flow to irrational CFT
- Generalization to other dimensions

Appendix

Commutativity

$\mathbb{Z}_2$ gauging and relevant deform. w/singlets commute.

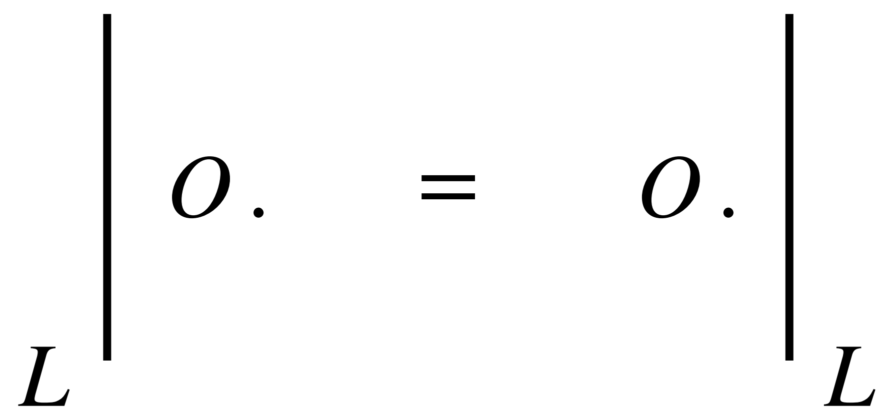
For singlet,



Commutativity

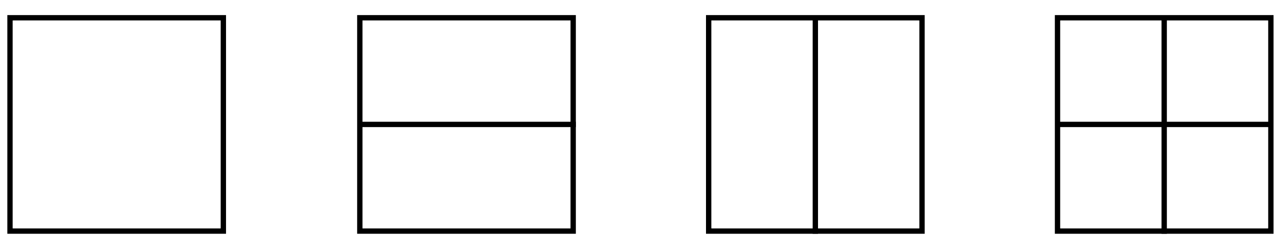
$\mathbb{Z}_2$ gauging and relevant deform. w/singlets commute.

For singlet,



[2207.06433 (KK)]

$$\left(Z_{T/\mathbb{Z}_2}\right)' \equiv \frac{1}{2}\left(Z_T[0,0]+Z_T[0,1]+Z_T[1,0]+Z_T[1,1]\right)'$$



$$= \frac{1}{2}\left\{(Z_T)'[0,0]+(Z_T)'[0,1]+(Z_T)'[1,0]+(Z_T)'[1,1]\right\} \equiv (Z_T)'_{/\mathbb{Z}_2}$$

Another side of our proposal

Conjecture: for $m = 2M + 1 \ (M = 2,3,\ldots)$,

there is no MTC satisfying 3 conditions simultaneously:

1) which has rank $2M < r < M(2M - 1)$;

2) which has central charge $c < 1 - \dfrac{3}{(2M+1)(M+1)}$;

3) which contains surviving sym. category.