Integrable three-point functions in ABJM theory

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Based on arxiv:2103.15840, 2103.16580, 2208.12010

BIMSA HEP-TH Seminar-September 14th

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- Field Content: Two vector fields $A_\mu, \hat{A}_\mu$, four complex scalar fields Y^A, and four Dirac fermions ψ_A.
- Global Symmetry: $SU(4)_R \subset OSP(6|4)$

Gauge Invariant Field Operators

In general, we can construct the gauge invariant multi-trace operators

$$\mathrm{tr}(X_{i_1}\bar{X}_{i_2}\cdots)\cdots\mathrm{tr}(X_{j_1}\bar{X}_{j_2}\cdots) \tag{1}$$

where the fields at the odd position are in bifundamental representation $(N,\bar{N})$ of the gauge group and the fields at the odd position are in anti-bifundamental representation $(\bar{N},N)$ of the gauge group.

Single Trace Operators

In the planar limit, the most important operators are single trace operators. We can find a unique state with the highest weight $\mathrm{tr}(Y^1\bar{Y}_4)^L$, the more complicated fields can be generated by changing Y^1,$\bar{Y}_4$ to other fields. These single trace operators can be described as an alternating spin chain which has the corresponding ground state $\mathrm{tr}(Y^1\bar{Y}_4)^L$. The ground state preserves $SU(2|2)\times U(1)$ symmetry.

Fact 1

ABJM theory is integrable in the planar limit [J. A. Minahan K. Zarembo 2008], the dilatation operators can be thought of as the Hamiltonian of an integrable OSP(6|4) spin chain.

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Fact 2

The integrable Hamiltonian can be diagonalized via Bethe Ansatz.

Closed Sector

In general, we can construct closed subsector by finding a linear combination T of some conserved charges that is positive semi-definite on the field multiplets. Then the closed subsector contains all fields which has vanishing linear combination charge.

There exists a scalar sector that only contains the scalar fields $Y, \bar{Y}$. It is a closed subsector for the two loop Hamiltonian.

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BPS Operators-Single Trace

The single-trace $\frac{1}{3}$ BPS operators in ABJM theory can be written as

$$O_L^{\circ}(x;n,\bar{n}) \equiv \mathrm{tr}\left[\left((n\cdot Y)(\bar{n}\cdot \bar{Y})\right)^L\right] \tag{2}$$

where

$$n\cdot Y \equiv \sum_{I=1}^{4} n_I Y^I \qquad \bar{n}\cdot \bar{Y} \equiv \sum_{I=1}^{4} \bar{n}^I \bar{Y}_I \tag{3}$$

$$n\cdot \bar{n} = 0 \tag{4}$$

BPS Operators-Subdeterminant Operator

Another $\frac{1}{3}$ BPS operator is Giant Graviton which can be expressed as

$$\mathcal{D}_M(x;n,\bar{n}) \equiv \frac{1}{M!}\delta^{[b_1\cdots b_M]}_{[a_1\cdots a_M]}[(n\cdot Y)(\bar{n}\cdot\bar{Y})]^{a_1}_{b_1}\cdots[(n\cdot Y)(\bar{n}\cdot\bar{Y})]^{a_M}_{b_M} \tag{5}$$

with $n\cdot\bar{n}=0$

$$\delta^{[b_1\cdots b_M]}_{[a_1\cdots a_M]} \equiv \sum_{\sigma\in S_M}(-1)^{|\sigma|}\delta^{b_1}_{a_{\sigma_1}}\cdots\delta^{b_M}_{b_{\sigma_M}} \tag{6}$$

The operator with maximal R charge (M=N) is called maximal Giant Graviton.

Generating Function

It is convenient to define a generator function of Giant Graviton:

$$\mathcal{G}(x;n,\bar{n},t) \equiv \det \begin{pmatrix} 1 & -t(\bar{n}\cdot\bar{Y}) \\ t(n\cdot Y) & 1 \end{pmatrix} = \det\left[1+t^2(n\cdot Y)(\bar{n}\cdot\bar{Y})\right] \qquad (7)$$

The Giant Graviton with fixed charge can be extracted through it.

$$\mathcal{D}_M(x;n,\bar{n}) = \oint \frac{dt}{2\pi i t^{1+2M}}\,\mathcal{G}(x;n,\bar{n},t) \qquad (8)$$

Structure Constants and Integrability

At weak coupling, the structure constants of two subdeterminant operators and one non-BPS operator can be computed by Wick contraction after mapping the operators to the spin chain language. [Y. Jiang, S. Komatsu and E. Vescovi,19][G. Chen, R. de Mello Koch, M. Kim and H. J. Van Zyl,19]

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1. Two subdetermiant operators can be written as a integrable boundary state by using effective field theory.
2. The non-BPS operator is generated by Coordinate Bethe Ansatz or Nested Bethe Ansatz

Effective Field Theory

The method proposed in [Y. Jiang, S. Komatsu and E. Vescovi,19] can be generalized to ABJM theory.

Consider the following correlation function:

$$G_m = \left\langle \left(\prod_{j=1}^{m} \mathcal{G}_j \right) \mathcal{O}(y) \right\rangle \tag{9}$$

where $\mathcal{G}_j$ is a generating function for Giant Gravitons.

Extracting the revelent terms will get the subdeterminant three point function.

$$\left\langle \left(\prod_{j=1}^{m} \mathcal{D}_{M_j} \right) \mathcal{O}(y) \right\rangle = \left(\prod_{j=1}^{m} \oint \frac{dt_j}{t_j^{1+2M_j}} \right) G_m \tag{10}$$

Since giant gravitons carry large R-charges, most of these integrals in the large N limit can be computed by the saddle-point approximation.

$$G_m = \frac{1}{Z_Y}\int DY^I D\bar{Y}_I \left(\prod_{j=1}^{m}\mathcal{G}_j\right) O \exp\left[-\frac{k}{4\pi}\int d^3x tr(\partial_\mu \bar{Y}_I \partial^\mu Y^I)\right] \tag{11}$$

with

$$Z_Y = \int DY^I D\bar{Y}_I \exp\left[-\frac{k}{4\pi}\int d^3x tr(\partial_\mu \bar{Y}_I \partial^\mu Y^I)\right] \tag{12}$$

The above integral can be written as an overlap between a spin chain state and a matrix product state in the large N limit.

Result

The three-point functions for maximal giant graviton can be written as the following expression:

$$\mathfrak{D}_{N|O} = -\frac{(-i)^J}{2^{\Delta-J}}\frac{\langle\mathcal{B}|O\rangle}{\sqrt{L\langle O|O\rangle}}, \tag{13}$$

where the state $\langle\mathcal{B}|$ is defined by

$$\langle\mathcal{B}| \equiv \sum_{A_s,B_s=1,4}\langle A_1\bar{B}_1\cdots A_L\bar{B}_L|\left(1+(-1)^J\right). \tag{14}$$

Here J is the $U(1)$ R-charge

$$J = L - \text{number of 4 on odd sites} - \text{number of } \bar{1} \text{ on even sites.} \tag{15}$$

Tree Level Computation

The structure constants can be calculated at tree level. The single trace operators should be the eigenstate of two loop dilatation operator. The tree level dilatation has huge degeneracies and the leading correction to the dilatation operator in ABJM theory is two loop correction.

The spectrum of two loop dilatation operator can be mapped to a spin chain model, which is shown to be integrable.[D. Bak and S.-J. Rey,2008]J. [A. Minahan, W. Schulgin and K. Zarembo,2009].

Two-Loop ABJM Hamiltonian

The dilatation operator in the scalar sector of ABJM theory is described by the $SU(4)$ alternating spin chain. The Hamiltonian is

$$H = \frac{\lambda^2}{2} \sum_{l=1}^{2L} (2 - 2P_{l,l+2} + P_{l,l+2}K_{l,l+1} + K_{l,l+1}P_{l,l+2}) \quad (16)$$

where $P_{i,j}, K_{i,j}$ denote the permutation operator and trace operator act on the i-th and j-th sites. Let $|i\rangle$ denote the Hilbert space on the spin chain, P,K act as

$$P|i\rangle \otimes |j\rangle = |j\rangle \otimes |i\rangle, \quad K|i\rangle \otimes |j\rangle = \delta_{ij} \sum_{i=1}^{4} |k\rangle \otimes |k\rangle. \quad (17)$$

Bethe Ansatz

The two loop Hamiltonian is integrable and can hence be solved by Bethe Ansatz.

The $SU(4)$ spin chain has four R matrices

$$\begin{aligned} R_{ab}(u) &= u\mathrm{I}_{ab} + P_{ab} \\ R_{\bar{a}\bar{b}}(u) &= u\mathrm{I}_{ab} + P_{\bar{a}\bar{b}} \\ R_{a\bar{b}}(u) &= -(u+2)\mathrm{I}_{ab} + K_{a\bar{b}} \\ R_{\bar{a}b}(u) &= -(u+2)\mathrm{I}_{ab} + K_{\bar{a}b}, \end{aligned} \tag{18}$$

where a and $\bar{a}$ denote the fundamental/anti-fundamental representation of $SU(4)$.

We can define two different monodromy matrices by introducing two different reprenstations of auxiliary spaces.

$$T_a(u) = R_{a1}(u)R_{a\bar{1}}(u)\cdots R_{aL}(u)R_{a\bar{L}}(u)$$
$$T_{\bar{a}}(u) = R_{\bar{a}1}(u)R_{\bar{a}\bar{1}}(u)\cdots R_{\bar{a}L}(u)R_{\bar{a}\bar{L}}(u). \tag{19}$$

Taking the trace of the auxiliary spaces gives the transfer matrices

$$\tau(u) = \mathrm{tr}_a T_a(u), \quad \bar{\tau}(u) = \mathrm{tr}_{\bar{a}} T_{\bar{a}}(u). \tag{20}$$

Two Hamiltonians can be generated from the transfer matrices

$$H_1 = (\tau(0))^{-1}\frac{d}{du}\tau(u)|_{u=0}, \quad H_2 = (\bar{\tau}(0))^{-1}\frac{d}{du}\bar{\tau}(u)|_{u=0}. \tag{21}$$

Then the two-loop anomalous dimension matrix of scalar sector of ABJM theory can be mapped to the Hamiltonian $H = H_1 + H_2$ (up to the rescale of the Hamiltonian and the a constant shift).

Vacuum and Excitation

The vacuum of spin chain is $\mathrm{tr}[(Y^1\bar{Y}_4)^L]$. The general Bethe state is described by two sets of momentum carrying Bethe roots $\boldsymbol{u}, \boldsymbol{v}$ and the auxiliary Bethe roots $\boldsymbol{w}$.

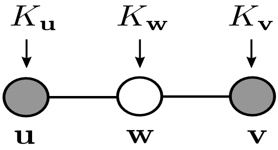

Figure 1

Bethe Equations

$$1 = e^{i\phi_{u_j}} = \left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}}\right)^L \prod_{\substack{k=1 \\ k\neq j}}^{K_\mathrm{u}} S(u_j, u_k) \prod_{k=1}^{K_\mathrm{w}} \tilde{S}(u_j, w_k) \tag{22}$$

$$1 = e^{i\phi_{w_j}} = \prod_{\substack{k=1 \\ k\neq j}}^{K_\mathrm{w}} S(w_j, w_k) \prod_{k=1}^{K_\mathrm{u}} \tilde{S}(w_j, u_k) \prod_{k=1}^{K_\mathrm{v}} \tilde{S}(w_j, v_k) \tag{23}$$

$$1 = e^{i\phi_{v_j}} = \left(\frac{v_j + \frac{i}{2}}{v_j - \frac{i}{2}}\right)^L \prod_{\substack{k=1 \\ k\neq j}}^{K_\mathrm{v}} S(v_j, v_k) \prod_{k=1}^{K_\mathrm{w}} \tilde{S}(v_j, w_k), \tag{24}$$

and

$$S(u,v) = \frac{u - v - i}{u - v + i}, \quad \tilde{S}(u,v) = \frac{u - v + \frac{i}{2}}{u - v - \frac{i}{2}}. \tag{25}$$

Eigenvalue

The eigenvalues of the transfer matrices can be obtained from the nested Bethe ansatz.

$$
\begin{aligned}
\Lambda(u) =&(-u-2)^L(u+1)^L\prod_{i=1}^{K_u}\frac{u-iu_i-1/2}{u-iu_i+1/2} \\
&+(-u)^L(u+1)^L\prod_{i=1}^{K_v}\frac{u-iv_i+5/2}{u-iv_i+3/2} \\
&+(-u)^L(u+2)^L\prod_{i=1}^{K_u}\frac{u-iu_i+3/2}{u-iu_i+1/2}\prod_{i=1}^{K_w}\frac{u-iw_i}{u-iw_i+1} \\
&+(-u)^L(u+2)^L\prod_{i=1}^{K_v}\frac{u-iv_i+1/2}{u-iv_i+3/2}\prod_{i=1}^{K_w}\frac{u-iw_i+2}{u-iw_i+1},
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\bar{\Lambda}(u) =&(-u)^L(u+1)^L\prod_{i=1}^{K_u}\frac{u-iu_i+5/2}{u-iu_i+3/2} \\
&+(-u-2)^L(u+1)^L\prod_{i=1}^{K_v}\frac{u-iv_i-1/2}{u-iv_i+1/2} \\
&+(-u)^L(u+2)^L\prod_{i=1}^{K_u}\frac{u-iu_i+1/2}{u-iu_i+3/2}\prod_{i=1}^{K_w}\frac{u-iw_i+2}{u-iw_i+1} \\
&+(-u)^L(u+2)^L\prod_{i=1}^{K_v}\frac{u-iv_i+3/2}{u-iv_i+1/2}\prod_{i=1}^{K_w}\frac{u-iw_i}{u-iw_i+1}.
\end{aligned}
\tag{27}
$$

Tree level result-Selection rules

As a result of the numerical experiments, we found a set of selection rules in order for the overlap to be non-zero.

The first and the most obvious selection rule is

0. The numbers of Bethe roots for each node need to satisfy

$$K_{\mathrm{u}} = K_{\mathrm{v}} = K_{\mathrm{w}}, \tag{28}$$

This simply follows from the fact that $\langle B_\theta|$ only contains fields Y^1, Y^4, $\bar{Y}_1$ and $\bar{Y}_4$: In order for $|\mathrm{u}, \mathrm{w}, \mathrm{v}\rangle$ to contain kets involving Y^1, Y^4, $\bar{Y}_1$, $\bar{Y}_4$ only, we need to set $K_{\mathrm{u}} = K_{\mathrm{v}} = K_{\mathrm{w}}$.

Pair Structure

In addition to this constraint, the overlap also obeys the following constraints:

1. The $U(1)$ R-charge of the state, $J = L - \frac{K_{\mathrm{u}}+K_{\mathrm{v}}}{2}$, must be even.
2. The rapidities of the right node must be (-1) times the rapidities of the left node:

$$\boldsymbol{v} = -\boldsymbol{u}, \tag{29}$$

3. The rapidities of the middle node should be parity-symmetric, namely

$$\boldsymbol{w} = \begin{cases} (w_1, -w_1, w_2, -w_2, \ldots) & K_w : \text{even}, \\ (w_1, -w_1, w_2, -w_2, \ldots, 0) & K_w : \text{odd}. \end{cases} \tag{30}$$

Main Formula I

The relvent Gaudin determinant is

$$G = \begin{pmatrix} \partial_{u_i}\phi_{u_j} & \partial_{u_i}\phi_{w_j} & \partial_{u_i}\phi_{v_j} \\ \partial_{w_i}\phi_{u_j} & \partial_{w_i}\phi_{w_j} & \partial_{w_i}\phi_{v_j} \\ \partial_{v_i}\phi_{u_j} & \partial_{v_i}\phi_{w_j} & \partial_{v_i}\phi_{v_j} \end{pmatrix}, \tag{31}$$

where ϕ's was defined through the Bethe ansatz equations.

The Gaudin determinant factorize into detG^+detG^- when we use the pair structure.

Main Formula II

The structure constant for maximal giant graviton and non-BPS operators gives

$$\mathfrak{D}_{N|O} = \frac{i^J + (-i)^J}{2^{\Delta - J}} \sqrt{\prod_{j=1}^{K_{\mathrm{u}}} \left(u_j^2 + \frac{1}{4}\right) \prod_{k=1}^{\lceil \frac{K_{\mathrm{w}}}{2} \rceil} \frac{1}{w_k^2 (w_k^2 + \frac{1}{4})} \frac{\det G_+}{\det G_-}}. \tag{32}$$

Summary I

Boundary State

EFT translate the determinat operator into a boundary state $\sum_{A,B=1,\cdots 4}\langle AB\cdots|(1+(-1)^J)$

CBA and ABA

A general single trace operator can be constructed from Bethe ansatz $|u,v,w\rangle = \sum \underbrace{\Psi}_{\text{wave function}} |state\rangle$.

Selection Rules

The non-vanishing overlap requires Bethe roots to satisfy some selection rules.

1. $\boldsymbol{u} = -\boldsymbol{v}, \boldsymbol{w} = -\boldsymbol{w}$
2. J is even, number of Bethe roots is equal.

Summary II

Overlap

The final has an universal integrable structure

$$\mathfrak{D}_{N|O} = \text{prefactor} * \underbrace{\sqrt{\frac{G^+}{G^-}}}_{\text{universal part}} \tag{33}$$

where G^+, G^- are the subdeterminants of the Guadin determinat.

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Integrable Boundary Condition

Integrabel Boundary

In [L. Piroli, B. Pozsgay, E. Vernier, 2017], the author proposes the integrable boundary condition

$$\tau(u)|\Psi\rangle = \Pi\tau(u)\Pi|\Psi\rangle \tag{34}$$

where $\Pi|ijkl\rangle = |lkji\rangle$.
Subsequently, [T. Gombor, Z. Bajnok, 2020] refine this definition and generalize it by considering the allowed pair structure. For ABJM theory, we have two integrable boundaries:

$$\text{chiral}: \tau(u)|\Psi\rangle = \Pi\tau(u)\Pi|\Psi\rangle, \tag{35}$$
$$\text{achiral}: \tau(u)|\Psi\rangle = \Pi\bar{\tau}(u)\Pi|\Psi\rangle \tag{36}$$

We can remove the factor $1+(-1)^J$ in the boundary state if we focus on the first selection rule. Then we have the boundary state

$$|\Psi\rangle = \sum_{B_s,\bar{B}_s=1,4} |B_1\bar{B}_1\cdots B_L\bar{B}_L\rangle. \tag{37}$$

Integrable Boundary

The boundary state $|\Psi\rangle$ satisifies the achiral integrable condition

$$\tau(u)|\Psi\rangle = \Pi\bar{\tau}(u)\Pi|\Psi\rangle \tag{38}$$

[T. Gombor, 2021] prove this integrable condition for (37) by using the K matrix method.

Result

The LHS of (38) is equivalent to

$$\Pi \mathrm{tr}_{\bar{a}}(R_{\bar{a}1}(u)R_{\bar{a}\bar{1}}(u)\cdots R_{\bar{a}L}(u)R_{\bar{a}\bar{L}}(u))|\bar{B}_L B_L \cdots \bar{B}_1 B_1\rangle. \tag{39}$$

The crossing symmetry relate the transpose of the auxiliary space of R matrices to the anti-fundamental R matrices.

$$R_{\bar{a}i}(u) = R_{ai}^{t_0}(-u-2). \tag{40}$$

Then we have

$$\Pi \mathrm{tr}_a(R_{a\bar{L}}(-u-2)R_{aL}(-u-2)\cdots R_{a\bar{1}}(-u-2)R_{a1}(-u-2))|\bar{B}_L B_L \cdots \bar{B}_1 B_1\rangle$$
$$= \mathrm{tr}_a(R_{a1}(-u-2)R_{a\bar{1}}(-u-2)\cdots R_{aL}(-u-2)R_{a\bar{L}}(-u-2))|B_1 \bar{B}_1 \cdots B_L \bar{B}_L\rangle.$$

The integrable boundary condition becomes

$$\tau(-u-2)|\Psi\rangle = \tau(u)|\Psi\rangle \tag{41}$$

The overlap between the boundary state and the Bethe state gives

$$\langle\Psi|\tau(u)|\mathbf{u,v,w}\rangle = \langle\Psi|\Pi\bar{\tau}(u)\Pi|\mathbf{u,v,w}\rangle = \langle\Psi|\tau(-u-2)|\mathbf{u,v,w}\rangle. \tag{42}$$

The pair structure can be obtained from (42)

$$\Lambda(u) = \Lambda(-u-2) \Rightarrow \mathbf{u} = -\mathbf{v}, \mathbf{w} = -\mathbf{w}. \tag{43}$$

Proof: Notation

It is useful to write down the indices experssion of P, K, I:

$$I^{aA}_{bB} = \delta^a_b \delta^A_B, \quad P^{aA}_{bB} = \delta^a_B \delta^A_b, \quad K^{aA}_{bB} = \delta^{aA} \delta_{bB}, \tag{44}$$

Example: Acting P, K on the Hilbert space, we obtain

$$P_{ij}|ij\rangle = |ji\rangle \leftrightarrow \delta^a_j \delta^A_i \quad \text{(components)}$$

$$K_{ij}|ij\rangle = \delta_{ij} \sum_{k=1}^{4} |kk\rangle \leftrightarrow \delta_{ij} \delta^{aA} \quad \text{(components)}$$

Proof

(41) can be written as

$$\sum_{\bar{B}_i, B_i=1,4} \mathrm{tr}[(uI+P)(-(u+2)I+K)\cdots]|B_1\bar{B}_1\cdots\rangle = \sum_{\bar{B}_i, B_i=1,4} \mathrm{tr}[(-(u+2)I+P)(uI+K)\cdots]|B_1\bar{B}_1\cdots\rangle. \quad (45)$$

It is useful to give the explicit form of the contractions of P, K, I

$$(K)^{a\bar{A}_1}_{b\bar{B}_1}(P)^{bA_2}_{cB_2} = \delta^{a\bar{A}_1}\delta_{b\bar{B}_1}\delta^{b}_{B_2}\delta^{A_2}_{c} = \delta_{\bar{B}_1,B_2}\delta^{a\bar{A}_1}\delta^{A_2}_{c} \quad (46)$$

$$(P)^{aA_1}_{bB_1}(K)^{b\bar{A}_2}_{e\bar{B}_2} = \delta^{a}_{B_1}\delta^{b\bar{A}_2}\delta^{b}_{A_1}\delta_{e\bar{B}_2} = \delta^{a}_{B_1}\delta^{A_1\bar{A}_2}\delta_{e\bar{B}_2} \quad (47)$$

$$(K)^{a\bar{A}_1}_{b\bar{B}_1}(K)^{b\bar{A}_2}_{c\bar{B}_2} = \delta^{a\bar{A}_1}\delta_{\bar{B}_1 b}\delta^{\bar{A}_2 b}\delta_{c\bar{B}_2} = \delta^{a\bar{A}_1}\delta^{\bar{A}_2}_{\bar{B}_1}\delta_{c\bar{B}_2} \quad (48)$$

$$(P)^{aA_1}_{bB_1}(P)^{bA_2}_{cB_2} = \delta^{a}_{B_1}\delta^{A_2}_{c}\delta^{A_1}_{b}\delta^{b}_{B_2} = \delta^{a}_{B_1}\delta^{A_2}_{c}\delta^{A_1}_{B_2}. \quad (49)$$

Contraction

Properties of Contraction

PP/KK contractions involve $\delta^{A_i}_{B_j}$ that means the permutation between the odd/even sites.
P_iK_j contractions require to sum over the indices on i, j sites.
K_iP_j contractions require the initial indices of i, j sites are equal.

A key observation is that the coefficients in front of $\mathrm{tr}(PI\cdots IK)$ on both sides of (45) are

$$\text{coefficients of}\quad \mathrm{tr}(PI\cdots IK) = (-u^2-2u)^{(\text{number of I})/2}. \tag{50}$$

This inspired us to divide the whole spin chain into different intervals separated by various $PI\cdots IK$ factors.

Figure 2: Spin Chain

Step 1

The expansion of the transfer matrix can be divided into two parts

transfer matrix = terms containing $PI\cdots IK$ + terms not containing $PI\cdots IK$.

1. Terms not containing PK factor mean these terms only contain PI/IK, then we can prove

$$
\begin{aligned}
&\mathrm{tr}(\cdots \underbrace{P_l I_l}_{l\text{th factor}} \cdots \underbrace{P_i I_i}_{i\text{th factor}} \cdots \underbrace{P_j I_j}_{j\text{th factor}} \cdots)|\Psi\rangle \quad \text{in LHS/RHS} = \\
&\mathrm{tr}(\cdots \underbrace{I_l K_l}_{l\text{th factor}} \cdots \underbrace{I_i K_i}_{i\text{th factor}} \cdots \underbrace{I_j K_j}_{j\text{th factor}} \cdots)|\Psi\rangle \quad \text{in RHS/LHS.}
\end{aligned} \tag{51}
$$

Step 1

$\mathrm{tr}(PI\cdots PI)/\mathrm{tr}(IK\cdots IK)$ involves the permutation between the indices at the position of P/K.

Example:

$$\mathrm{tr}(PIPI)|\Psi\rangle = \sum_{B_i,\bar{B}_i=1,4} |B_2\bar{B}_1B_1\bar{B}_2\rangle,$$
$$\mathrm{tr}(IKIK)|\Psi\rangle = \sum_{B_i,\bar{B}_i=1,4} |B_1\bar{B}_2B_2\bar{B}_1\rangle,$$

Our boundary state is invariant under the permutation of the indices at the odd/even sites, so we have the correspondence (51).

Step 2

2. Due to the trace on the auxiliary space, the end-to-end KP factor has the same index structure of the PK factor.

A general terms containing the PK facor has the following form

$$\underbrace{PI\cdots IK}_{\text{PK factor}} II\cdots \underbrace{IK\cdots IK\cdots}_{\text{number of K}=m} II\cdots \underbrace{PI\cdots PI\cdots}_{\text{number of P}=n} II\cdots \underbrace{PI\cdots IK}_{\text{PK factor}}. \quad (52)$$

We claim the following correspondence in (45)

$$\mathrm{tr}(\cdots \underbrace{PI\cdots IK}_{\text{PK factor}} \underbrace{II\cdots}_{\text{only I}} \underbrace{IK\cdots IK\cdots}_{\text{number of K}=m} \underbrace{II\cdots}_{\text{only I}} \underbrace{PI\cdots PI\cdots}_{\text{number of P}=n} \underbrace{II\cdots}_{\text{only I}} \underbrace{PI\cdots IK}_{\text{PK factor}} \cdots)|\Psi\rangle \leftrightarrow$$

$$\mathrm{tr}(\cdots \underbrace{PI\cdots IK}_{\text{PK factor}} \underbrace{II\cdots}_{\text{only I}} \underbrace{IK\cdots IK\cdots}_{\text{number of K}=n} \underbrace{II\cdots}_{\text{only I}} \underbrace{PI\cdots PI\cdots}_{\text{number of P}=m} \underbrace{II\cdots}_{\text{only I}} \underbrace{PI\cdots IK}_{\text{PK factor}} \cdots)|\Psi\rangle.$$

The number of IK/PI is constrained by requiring the match of the coefficient.

Step 2

The position of PI, IK factors in the interval of LHS/RHS are labelled by $\mathcal{C}/\mathcal{D}$. We require $\mathcal{C} = \mathcal{D}$.

Example:

$$\mathrm{tr}\Big(\underbrace{PIIK}_{\text{PK factor}}\ \underbrace{IK_i\cdots}_{\text{number of K}=1}\ \underbrace{PI_j\cdots PI_k\cdots PI_l}_{\text{number of P}=3}\ \underbrace{PIIK}_{\text{PK factor}}\Big)|\Psi\rangle \leftrightarrow \tag{53}$$

$$\mathrm{tr}\Big(\underbrace{PIIK}_{\text{PK factor}}\ \underbrace{IK_i\cdots IK_j\cdots IK_k}_{\text{number of K}=3}\ \underbrace{PI_l\cdots}_{\text{number of P}=1}\ \underbrace{PIIK}_{\text{PK factor}}\Big)|\Psi\rangle.$$

Here the same color means the same site on the spin chain and the IK_i, PI_j are marked by their sites on the spin chain, hence we have $\mathcal{C} = \mathcal{D} = \{i, j, k, l\}$.

Step 2

In general, there are three types of Kronecker delta function:

1. $\delta^{A_i}_{B_j}$ from PP/KK contractions, induce the permutation among the odd/even sites.

2. $\delta_{B_o\bar{B}_p}$ from KP contractions, Summing over the original index B gives us a const,

$$\sum_{\bar{B}_i, B_i=1,4} \delta_{B_o\bar{B}_p}\delta_{B_m\bar{B}_m}\cdots = 2^n, \tag{54}$$

where n is the number of $IKI\cdots IPI$ contraction.

3.$\delta^{A_i\bar{A}_j}$: $\delta^{A_i\bar{A}_j}$ from the PK contractions denote the sum of the final state

$$\delta^{A_i\bar{A}_j} \leftrightarrow \sum_{\tilde{A}=1,2,3,4} |\cdots \underbrace{\tilde{A}}_{i-thsite} \cdots \underbrace{\tilde{A}}_{j-thsite} \rangle. \tag{55}$$

Step 2

Because our boundary state is invariant under the permutation of the indices on the odd/even sites and the number of KP contraction is equal to the number of the PK contraction, Both sides of (53) will give the same result once we fix the position of the PK factor.

Hence, both sides of (53) are

$$2^n \sum_{\substack{B_i,\bar{B}_i\cdots=1,4 \\ \tilde{A}_i=1,2,3,4}} |\cdots\tilde{A}_1\bar{B}_iB_j\tilde{A}_1B_k\bar{B}_l\cdots B_m\bar{B}_n\tilde{A}_2\bar{B}_oA_p\tilde{A}_2\cdots\tilde{A}_3\bar{B}_rB_s\tilde{A}_3\cdots\rangle. \quad (56)$$

Applying (51),(53) to every PK factor and its interval, we get the desired correspondence (45).

More General State

Integrable Boundary State

The same analysis can be applied to more general state,

$$|\Psi'\rangle = \sum_{B_i,\bar{B}_i=1,2,3,4} \mathrm{tr}(t_{B_1}\bar{t}_{\bar{B}_1}\cdots t_{B_2}\bar{t}_{\bar{B}_2}\cdots)|B_1\bar{B}_1\cdots B_2\bar{B}_2\cdots\rangle, \tag{57}$$

where $t_i, \bar{t}_i$ commute to each other.

Summary

Integrable Boundary State

The boundary state

$$|\Psi'\rangle = \sum_{B_i,\bar{B}_i=1,2,3,4} \mathrm{tr}(t_{B_1}\bar{t}_{\bar{B}_1}\cdots t_{B_2}\bar{t}_{\bar{B}_2}\cdots)|B_1\bar{B}_1\cdots B_2\bar{B}_2\cdots\rangle, \tag{58}$$

is integrable in the sense of [L. Piroli, B. Pozsgay, E. Vernier, 2017].

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Previous Works

- The first attempts in $\mathcal{N} = 4\ SYM$[D. Bak, B. Chen and J.-B. Wu, 2011]. The mismatch between the extremal three point function in $\mathcal{N} = 4\ SYM$ and holographic compuatation.
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- Single particle basis [R. de Mello Koch and R. Gwyn,2004] [T. W. Brown,2007] [F. Aprile, J. Drummond, P. Heslop, H. Paul, F. Sanfilippo, M. Santagata and A. Stewart, 2020]

- Perfect match of the non-extremal three point functions between the $\mathcal{N} = 4$ SYM and holography.[P. Caputa, R. de Mello Koch and K. Zoubos, 2012]

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- Orbit average [Z. Bajnok, R. A. Janik and A. Wereszczy´nski, 2014]
- Regulation of the extremal three point functions. [H. Lin, 2012] [C. Kristjansen, S. Mori and D. Young, 2015]

AdS/CFT Setup

The AdS/CFT correspondence provides a map from field side to gravity side [Bissi, Kristjansen, Young, Zoubos, 2011].More precisely

- Field theory: (sub)determiant operators (heavy), single trace BPS operators (light).

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- gravity side (ABJM): (M theory background) $M5$ brane , supergravity fluctuations (light).

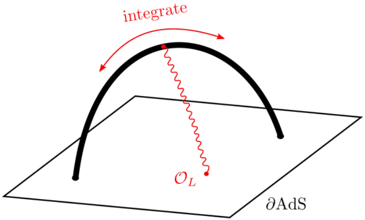

Figure 3: holographic duality

Wavefunction Contribution and Orbit Average

Two important effects were missed before.

- Orbit average: the semiclassical solution has a moduli space. Taking a average over the moduli space has already been applied in the computation of Heavy-Heavy-Light three point functions. [Z. Bajnok, R. A. Janik and A. Wereszczy´nski, 2014]

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- Wavefunction contribution: the contribution from the heavy operators.

A Toy Model

Consider a quantum mechanical system with a $U(1)$ global symmetry.

$$\theta \to \theta + c. \tag{59}$$

It is interesting to consider the expectation value of a light operator O with a large $U(1)$ charge $|J\rangle$, namely $\langle J|O(t=0)|J\rangle$, which is evaluated in the semi-classical (WKB) limit

$$J \to \infty, \qquad h \to 0, \qquad hJ: \text{ fixed}. \tag{60}$$

then the path integral can be written as

$$\langle J|O(t=0)|J\rangle = \int D\theta(t)\, e^{-iJ\theta(t=+\varepsilon)}\, O[\theta(t=0)] e^{iJ\theta(t=-\varepsilon)} e^{\frac{i}{h}S[\theta]}, \tag{61}$$

here $O[\theta(t=0)] = \langle\theta|O|\theta\rangle$ and we have used the relation $\langle\theta|O|\tilde{\theta}\rangle = \delta(\theta-\tilde{\theta})O[\theta(t=0)]$.

In the WKB limit, the path integral can be evaluated by saddle point approximation.

$$\frac{\delta S[\theta]}{\delta\theta(t)} + hJ\left(\delta(t+\varepsilon) - \delta(t-\varepsilon)\right) = 0. \tag{62}$$

The $U(1)$ invariance gives a family of solutions

$$\theta_c^*(t) \equiv \theta_0^*(t) + c, \qquad c \in [0, 2\pi]. \tag{63}$$

Taking the avarage over the moduli space gives

$$\langle J|O(t=0)|J\rangle \overset{\text{WKB}}{=} \int_0^{2\pi} \frac{dc}{2\pi}\, e^{-iJ\theta_c^*(+\varepsilon)} O[\theta_c^*(0)] e^{iJ\theta_c^*(-\varepsilon)} e^{\frac{i}{\hbar}S[\theta_c^*]}. \tag{64}$$

We assume the action is invariant under the translation.

$$S[\theta_c^*] = S[\theta_0^*]. \tag{65}$$

Therefore we obtain a simpler expression

$$\langle J|O(t=0)|J\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]} \int_0^{2\pi} \frac{dc}{2\pi}\, O[\theta_c^*(0)]. \tag{66}$$

This discussion can be generalized to $\langle J+q|O|J\rangle$.

$$\langle J+q|O(t=0)|J\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]} \int_0^{2\pi} \frac{dc}{2\pi}\, e^{-iqc}\, O[\theta_c^*(0)]. \tag{67}$$

In summary, there are two effects that will give the contribution to the VEV.

- First, when the bra and ket states are different, there is a nontrivial (boundary-term) contribution from the wave functions.

In summary, there are two effects that will give the contribution to the VEV.

- First, when the bra and ket states are different, there is a nontrivial (boundary-term) contribution from the wave functions.
- Second, the moduli space of saddle point equation should be averaged.

Using the radial quantization in conformal field theory, the correlation function can be expressed as

$$\text{CFT}: \qquad C_{\mathcal{D}_{M+k}\mathcal{D}_M O_L} = \langle \mathcal{D}_{M+k}|O_L(t=0)|\mathcal{D}_M\rangle. \tag{68}$$

The bra and ket can be mapped to the giant gravition with angular momentum $M+k$ and k on global AdS, and the O is correspondant to the supergravity fluctutation.

Collecting all the data, the three point functions can be determined by perturbing the DBI action and WZ action of the D-brane.

Evaluating the path integral of the worldvolume theory of the D branes.

$$\langle \hat{\mathcal{D}}_{M+k} | \hat{O}_L(t=0) | \hat{\mathcal{D}}_M \rangle = \int DX \, \Psi^*_{M+k}[X] \, \hat{O}_L[X(t=0)] \, \Psi_M[X] e^{-S_{\rm DBI+WZ}} , \tag{69}$$

The giant graviton carry two non-vanishing charges, the conformal dimension Δ and the $U(1)$ R-charge J. Applying the previous argument, two familises of solutions can be obtained by acting the generators $e^{-D\tau_0}$ and $e^{i\hat{J}\phi_0}$ to X^*_0.

$$X^*_{\tau_0,\phi_0} = X^*_0|_{t \to t - i\tau_0, \, \phi \to \phi + \phi_0} , \tag{70}$$

Here ϕ is the target space coordinate conjugate to the $U(1)$ rotation $\hat{J}$.

Since the wavefunction depend on the coordinates $\Psi \sim e^{-i\Delta t + iJ\phi}$, $\Psi^* \sim e^{i\Delta t - iJ\phi}$, the wavefunction contribution can be expressed as

$$\Psi \mapsto e^{-\Delta\tau_0} e^{iJ\phi_0} \Psi, \qquad \Psi^* \mapsto e^{+\Delta\tau_0} e^{-iJ\phi_0} \Psi^*. \tag{71}$$

Finally, the holographic computation in semiclassical limit is given by

$$\langle \hat{\mathcal{D}}_{M+k} | \hat{O}_L(t=0) | \hat{\mathcal{D}}_M \rangle \overset{\text{WKB}}{=} \underbrace{\int d\tau_0 \int \frac{d\phi_0}{2\pi}}_{\text{orbit average}} \\ \hat{O}_L[X^*_{\tau_0,\phi_0}(t=0)] \underbrace{e^{(\Delta_{M+k}-\Delta_M)\tau_0} e^{-i(J_{M+k}-J_M)\phi_0}}_{\text{wave function}}. \tag{72}$$

Schur polynomial

The correlation function involves two subdeterminant operators and one single trace BPS operator. The anti-symmetric Schur-polynomial operator $\chi_M(Z)$ which can be defined by a sub-determinant is given by

$$\mathcal{D}_M = \chi_M(Z) \equiv \frac{1}{M!}\delta^{[b_1 b_2 \cdots b_M]}_{[a_1 a_2 \cdots a_M]} Z^{a_1}_{b_1} \cdots Z^{a_M}_{b_M},$$
$$\delta^{[b_1 \cdots b_M]}_{[a_1 \cdots a_M]} \equiv \sum_{\sigma \in S_M} (-1)^{|\sigma|} \delta^{b_1}_{a_{\sigma_1}} \cdots \delta^{b_M}_{b_{\sigma_M}}. \quad (73)$$

and consider the single trace BPS operator

$$O_L \equiv \mathrm{tr}\tilde{Z}^L, \qquad \tilde{Z} = \frac{Z + \bar{Z} + Y - \bar{Y}}{2}, \quad (74)$$

Thanks to the supersymmetry, the structure constant is independent of the coupling constant.

Holography Realization

The diagonal three point functions $C_{\mathcal{D}_M \mathcal{D}_M O_L} = \langle \mathcal{D}_M | O_L | \mathcal{D}_M \rangle$ can be calculated by holography. Recalling the equation

$$\langle \hat{\mathcal{D}}_{M+k} | \hat{O}_L(t=0) | \hat{\mathcal{D}}_M \rangle = \int DX \, \Psi^*_{M+k}[X] \, \hat{O}_L[X(t=0)] \Psi_M[X] e^{-S_{\mathrm{DBI+WZ}}}, \tag{75}$$

Here we have $\hat{O}_L \sim \delta S_{DBI} + \delta S_{WZ}$.

The final result is

$$C_{\mathcal{D}_M \mathcal{D}_M O_L} = -\frac{i^L + (-i)^L}{2\sqrt{L}} \left(P_{\frac{L}{2}}(\cos 2\theta_0) + P_{\frac{L}{2}-1}(\cos 2\theta_0) \right). \tag{76}$$

where $cos\theta_0 = \frac{M}{N}$. This result perfectly matches with field theory caculation.

The result of off-diagonal three point functions reads

$$C_{\mathcal{D}_{M+k}\mathcal{D}_M O_L} = \int_{-\infty}^{\infty} d\tau_0 \int_0^{2\pi} \frac{d\phi_0}{2\pi} \hat{O}_L[X^*_{\tau_0,\phi_0}] e^{k\tau_0} e^{-ik\phi_0}, \tag{77}$$

Performing the holographic calculation, we get

$$\begin{aligned} &C_{\mathcal{D}_{M+k}\mathcal{D}_M O_L} = \\ &-\sqrt{L}\left(i^{L-k} + (-i)^{L-k}\right) \frac{\Gamma(\frac{L+k}{2}) \cos^2\theta_0 \sin^k\theta_0}{\Gamma(1+k)\Gamma(1+\frac{L-k}{2})} \\ &{}_2F_1\left(1+\tfrac{k-L}{2}, 1+\tfrac{k+L}{2}, 1+k; \sin^2\theta_0\right). \end{aligned} \tag{78}$$

This result perfectly matches with the field caculation of non-extremal cases.

Application in ABJM theory

The final result of ABJM theory is only non-vanishing even L and reads

$$C_{\mathcal{D}_M\mathcal{D}_M O_L} = \left(\frac{\lambda}{2\pi^2}\right)^{1/4}\frac{\sqrt{2L+1}}{L}(1+(-1)^L)\frac{\sqrt{\pi}\Gamma(\frac{L}{2}+1)}{\Gamma(\frac{L+3}{2})}(1-4\alpha^4)^{\frac{1}{2}(L-1)}$$

$$\times\Big[(1-4\alpha^4)\,{}_2F_1\left(-\tfrac{1}{2}(L+1),-\tfrac{L}{2};1;\tfrac{4\alpha^4}{4\alpha^4-1}\right) +2\alpha^4(L+1)\,{}_2F_1\left(-\tfrac{1}{2}(L-1),-\tfrac{L}{2}+1;2;\tfrac{4\alpha^4}{4\alpha^4-1}\right)\Big] \tag{79}$$

Because the three point functions of three BPS operators is not protected in ABJM theory, this result should be tested against integrability.

Extremal correlation function in $\mathcal{N} = 4$ SYM

Strong coupling caculation of extremal correlation function contains a term of the form (zero prefactor) $\times$ (divergent integral) in the extremal limit.

$$C_{\mathcal{D}_{M+k}\mathcal{D}_M O_L} = \int_{-\infty}^{\infty} d\tau_0\, e^{k\tau_0} \int_0^{2\pi} \frac{d\phi_0}{2\pi} e^{-ik\phi_0} \int_0^{2\pi} d\chi_3 \int_0^{\frac{\pi}{2}} d\chi_1 \left(I_{\text{finite}} + I_{\text{divergent}}\right) \tag{80}$$

I_{finite} is well-defined and finite even in the extremal limit and gives

$$C^{\text{finite}}_{\mathcal{D}_{M+L}\mathcal{D}_M O_L} = -\sqrt{L}(\cos\theta_0)^2(\sin\theta_0)^L. \tag{81}$$

Ambiguity of analytical continue

the integral of $I_{\mathrm{divergent}}$ is divergent in the extremal limit while it contains a prefactor which vanishes in the limit. One way to deal with this divergent term is to take a regularization. [C.Kristjansen, S.Mori and D.Young, 2015]

A better way to deal with this problem is to first consider a non-extremal three-point function $L-k>0$ and perform the analytic continuation to read off the result for the extremal limit $L-k=0$.

Ambiguity of analytical continue

The procedure above requires us to analytically continue $L-k$ from positive integers. However the analytic continuation is not unique. For instance, we can always add a term proportional to

$$\frac{\sin\pi(L-k)}{L-k}, \tag{82}$$

which vanishes for positive integer $L-k$ but changes the value at $L-k=0$. Therefore, the extremal correlation function has the ambiguity.

Appendix: Field computation

Consider the generating function of Giant Gravitons,

$$\mathcal{G}_j \equiv \det\left[1 + t_j(Y_j \cdot \Phi)\right](x_j), \tag{83}$$

where Y_j is a six-dimensional null vector and $\Phi \equiv (\Phi_1, \ldots, \Phi_6)$ are six real scalar fields in $\mathcal{N} = 4$ SYM.

The Giant Graviton with a fixed charge M can be extracted from the generating function:

$$\oint \frac{dt_j}{2\pi i t_j^{1+M}} \mathcal{G}_j. \tag{84}$$

Appendix: Field computation

Evaluating the correlation function of two generating functions and a BPS single-trace operator

$$O_L(x_3) \equiv \mathrm{tr}\left((Y_3 \cdot \Phi)^L\right)(x_3), \tag{85}$$

at tree level.

Using the effective field theory method, the correlation function can be written into the matrix product representation.

$$\langle O_L^S \rangle_\chi = -\oint_{|y|=1} \frac{dy}{2\pi i y} \mathrm{Tr}\left[\mathcal{T}^L\right], \tag{86}$$

Appendix: Field computation

Extracting the residue and combining everything, we arrive at the final result.

$$\langle O_L^S \rangle_\chi = -(2g^2)^{\frac{L}{2}} \left(\frac{d_{13}d_{23}}{d_{12}}\right)^{\frac{L}{2}} \frac{i^L + (-i)^L}{2} \left(P_{\frac{L}{2}}(\cos 2\theta_0) + P_{\frac{L}{2}-1}(\cos 2\theta_0)\right), \tag{87}$$

(87) leads to the following result for the structure constant

$$C_{\mathcal{D}_M \mathcal{D}_M O_L} = -\frac{i^L + (-i)^L}{2\sqrt{L}} \left(P_{\frac{L}{2}}(\cos 2\theta_0) + P_{\frac{L}{2}-1}(\cos 2\theta_0)\right). \tag{88}$$

This matches precisely with the result computed from D-branes.

Summary

- We apply the orbit average and wavefunction contribution to the holographic computation of $\mathcal{N} = 4$ SYM and ABJM theory.

Summary

- We apply the orbit average and wavefunction contribution to the holographic computation of $\mathcal{N} = 4$ SYM and ABJM theory.
- We perform the weak coupling caculations and find a perfect match between the field computation and the holographic computation.

Further Direction

1. Do various defects and operators correspond to an integrable boundary state?
2. Exact computation of the overlap.
3. Finite coupling description.
4. Further check by using superconformal bootstrap and perturbative computation.

Thank you!