Celestial Circle at BIMSA

Jingxiang

Resonances and Unitarity from Celestial Amplitude

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2205.14633

BIMSA

1. Quick review of celestial formalism

2. Motivation

3. Main example: $O(N)$ model

4. Results

5. Outlook

II Celestial Formalism

△ embedding formalism

nonlinear action of conformal group in CFT$_d$

$SO(d+1,1)$

can be realised as

linear action of Lorentz group in flat $\mathbb{R}^{d+1,1}$

Recall: projective null cone

$$\frac{\{X \in \mathbb{R}^{d+1,1},\ X^2 = 0\}}{X \sim \lambda X} = S^d$$

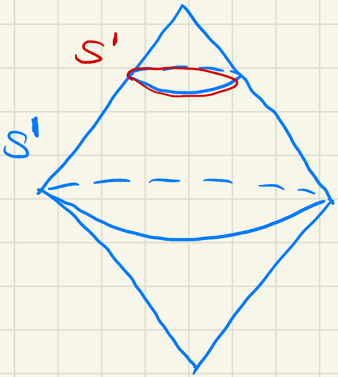
celestial formalism

amplitude in flat $\mathbb{R}^{d+1,1} \longrightarrow$ correlator in S^d.

[Pasterski, Shao, Strominger, ...]

Restrict to 3d massless scattering

amplitude in $\mathbb{R}^{1,2}$ $\longleftrightarrow$ conformal correlator in S^1



① massless momentum

$$\underbrace{\mathbb{R}^{1,2}} \ni \underbrace{p^\mu} = \pm\, \omega\,(1+x^2,\ 2x,\ 1-x^2)\ ,\quad \omega > 0$$

$x \in S^1$

② basis of states

momentum basis $\quad |p\rangle = e^{ip\cdot X}$ is the eigenstate of P_μ

labeled by momentum

conformal basis $\quad |\varphi_\Delta(x)\rangle$ is the scalar primary of $SO(2,1)$

$$\stackrel{\Delta}{=} \underbrace{\int_0^\infty d\omega\ \omega^{\Delta-1}}_{\text{Mellin}}\ \underbrace{e^{\pm ipX}}$$

$(z,\bar{z})$ z.

$\delta(z-\bar{z})$

③ amplitude:

$$\boxed{\tilde{A}(\Delta_i, x_i)} = \prod_{i=1}^{4} \int_0^\infty dw_i \; w_i^{\Delta_i - 1} \; \boxed{T(s,t)} \; \delta^{(3)}\left(\sum p_i\right)$$

$\Delta_i \to \Delta$

$$= \frac{1}{|x_{12}|^{2\Delta} |x_{34}|^{2\Delta}} \; 2^{-\Delta_T + 1} \frac{|z|}{\sqrt{|z-1|}} \int_0^\infty dw \; w^{\Delta_T - 4} \; T(s,t)$$

$f(z)$

where $s = \pm w_\Delta^2$, $t = -\frac{s}{z_\Delta}$, $u = \frac{1-z}{z} s$

$$z = \frac{x_{12} x_{34}}{x_{13} x_{24}}$$ cross ratio

One can verify:

$$\tilde{A}\left(\frac{az+b}{cz+d}\right) = \prod_i (cz_i + d)^{2\Delta} \; A(z_i)$$

Question:

1d $\subset$ CFT ?

2 Motivation.

① tree level $\longrightarrow$ loop level, finite coupling

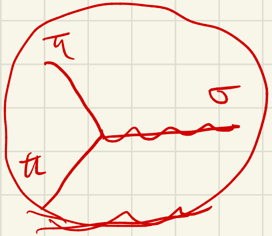
② resonance, unitarity

③ what is "CFT"?

④ 3d is easier

$\boxed{3}$ Main example: O(N) model [Coleman – Jackiw – Politzer]

$$\frac{1}{2}\partial_\mu \varphi^a \partial^\mu \varphi_a - \frac{1}{2}\mu_0^2 \varphi^a\varphi^a - \frac{\lambda_0}{8N}(\varphi^a\varphi^a)^2 \qquad \varphi^a,\ a=1,\cdots N$$



$$\langle\varphi\rangle^2 \neq 0 \quad\Rightarrow\quad O(N)\to O(N-1)$$

$\sigma = \varphi^N - \langle\varphi\rangle \rightsquigarrow$ massive

$\pi^a = \varphi^a,\ a=1,\cdots N-1 \rightsquigarrow$ massless a

$$T_4^{ab,cd}(p_1,p_2,p_3,p_4) = -\frac{\lambda}{N}\left[\frac{(p_{12})^2\,\delta^{ab}\delta^{cd}}{(p_{12})^2+\sqrt{(p_{12})^2}\lambda/16-2\mu^2} + \frac{(p_{13})^2\,\delta^{ac}\delta^{bd}}{(p_{13})^2+\sqrt{(p_{13})^2}\lambda/16-2\mu^2} + \frac{(p_{14})^2\,\delta^{ad}\delta^{bc}}{(p_{14})^2+\sqrt{(p_{14})^2}\lambda/16-2\mu^2}\right], \qquad (2.15)$$

$\Longrightarrow$

$$T_4^{ab,cd}\left(\omega^2, -\frac{\omega^2}{z}\right) = -\frac{\lambda}{N}\left[\frac{\omega^2\delta^{ab}\delta^{cd}}{\omega^2 + i\frac{\lambda}{16}\omega + 2\mu^2} + \frac{\omega^2\delta^{ac}\delta^{bd}}{\omega^2+\frac{\lambda}{16}\sqrt{z}\omega - 2\mu^2 z} + \frac{\omega^2\delta^{ad}\delta^{bc}}{\omega^2+\frac{\lambda}{16}\sqrt{\frac{z}{z-1}}\omega - 2\mu^2\frac{z}{z-1}}\right]. \qquad (3.17)$$

$\boxed{4}$ Results:

① Mellin transform $\alpha = \frac{\Delta_T - 3}{2}$

★ $f_s(z) = N N_\alpha \frac{z}{\sqrt{z-1}}\left[\delta^{ab}\delta^{cd} e^{i\pi\alpha} + \delta^{ac}\delta^{bd} z^\alpha + \delta^{ad}\delta^{bc}\left(\frac{z}{z-1}\right)^\alpha\right]$ $\quad z>1$

$\boxed{//}$ compare with tree level

[Lam Shao, 2017]

$\lambda \to 0$

$\Delta_T = \sum_i \Delta_i$

tree: $3 < \mathrm{Re}(\Delta_T) < 5$ $\quad \frac{3}{4} < \Delta_\phi < \frac{5}{4}$

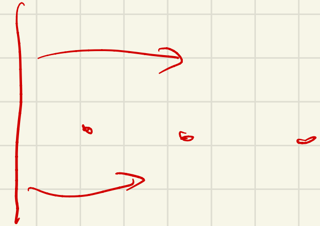
full: $1 < \mathrm{Re}(\Delta_T) < 3$

$\boxed{\Delta_\phi = \frac{1}{2} + i\mathbb{R}}$

$\boxed{4}$ Results

CPW

② CB expansion



$$\left\langle \phi_1(x_1)\ \phi_2(x_2) \,\middle|\, \phi_3(x_3)\ \phi_4(x_4) \right\rangle = \langle \phi\phi\phi\phi \rangle$$

$$\sum_{\psi} |\psi\rangle\langle\psi|$$

$$= \sum_{O\in\phi_1\times\phi_2} \sum_{\alpha = O,\, PO \ldots} \langle \phi_1\phi_2 | \alpha \rangle\langle \alpha | \phi_3\phi_4 \rangle$$

$$= \sum_{O} \frac{f_{12O}\, f_{34O}}{x_{12}^{2\Delta}\ x_{34}^{2\Delta}}\ \underbrace{g_\Delta(z)}_{\text{conformal block}}$$

OPE

$$k_\Delta(z) = z^\Delta\ {}_2F_1(\Delta, \Delta; 2\Delta, z)$$

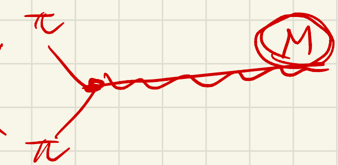
$$\frac{1}{N N_\alpha} f(z) = \underbrace{\sum_{n=0}^{\infty} \underbrace{B_I(n)}\, k_{\underbrace{2n+1}_{=\Delta_M}}(z)}_{\text{single trace}} + \underbrace{\sum_{n=0}^{\infty} C_I(n)\, k_{n+\alpha+1}(z)}_{\text{double trace}}$$

$$n+\alpha+1 \;\downarrow\; = \underbrace{2\Delta_\phi} + n - \frac{1}{2}$$

✓ ① $B_I(n) = \underbrace{f_{12\mathcal{O}}}\, f_{34\mathcal{O}}$

$$\underbrace{\sum_{M=1}} \frac{C(\Delta_\phi, \Delta_\phi, \Delta_M)^2}{D} \sim \underline{B_I(n)}$$

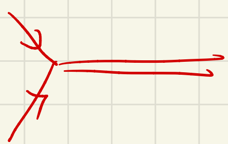
$$\Delta\, \underbrace{C(\Delta_\phi, \Delta_\phi, \Delta_M)} = \text{Mellin}\Big($$



$\Big)$

?? ② $C_I(n) \neq \sum C(\cdot\ \cdot\ \cdot)\, C(\cdot\ \cdot\ \cdot)$

[Kulp, Pasterski]



two particle states

5 Summary and Outlook

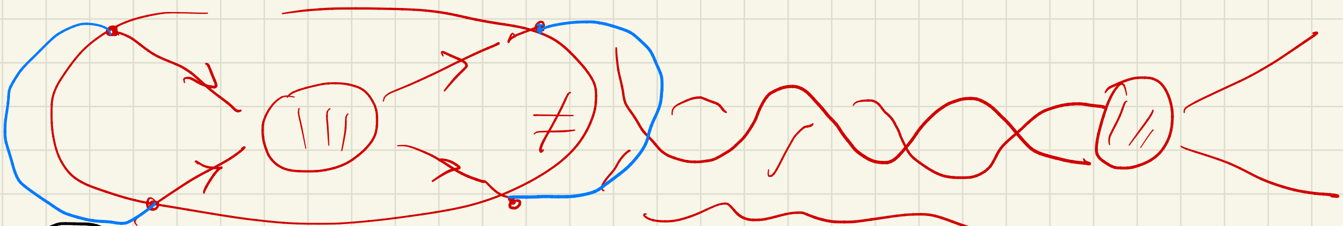
① resonances are important!

② factorization ?

③ Crossing Symmetry

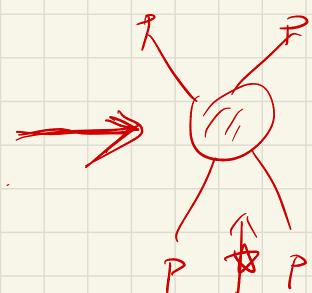
How to define amplitudes in gauge theories ?

Z 3d CSM

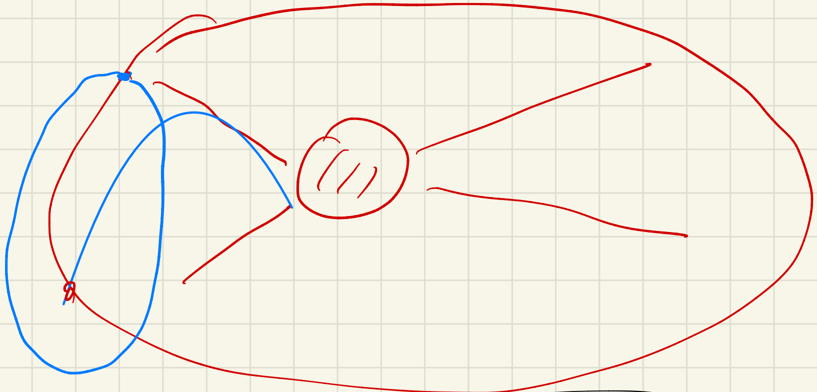


[Minwalla, Jain, ---

Gabai, Sandor, Yin.]



$$S_s = \cos(\pi\lambda)\ I + i\,\frac{\sin(\pi\lambda)}{\pi\lambda}\, T_s^{\text{trial}}$$



T u

4D / 2D

$\delta(z - \bar{z})$

[Hannesdottir, Mizera]
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