Orientifold Calabi-Yau Manifolds and Type IIB String Vacua

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Work with: Ross Altman, Jonathan Carifio, Brent Nelson, JHEP 03 (2022) 087
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Outline

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Flux Compactification I

From string to the real wold: 10D $\rightarrow$ 4D

What we want: $\mathcal{N} = 1$ Supersymmetry with chiral spectrum

Best under control: $\mathcal{N} = 1$ Flux Compactification

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Background Flux (in Type II):

- *Neveu-Schwarz flux:* $H_3 = dB_2, \ dH_3 = 0.$
- *Ramond flux:* $F_{p+1} = dC_p, \ dF_{p+1} = 0.$
- *Metric flux:* ${F_{ij}}^k$ from T-dual of H_{ijk}.
- *Non-geometric flux:* T-duality with Buscher rules.

$$H_{ijk} \xleftrightarrow{T_k} {F_{ij}}^k \xleftrightarrow{T_j} {Q_i}^{jk} \xleftrightarrow{T_i} R^{ijk} .$$

Flux Compactification II

Four dimensional $\mathcal{N} = 1$ supersymmetry flux compactification:

- Het string on CY_3
- Type IIA/B on CY_3 with orientifold (include Type I $\cong$ Type IIB orientifold with $O9-$plane) ✓
- F-theory on CY_4
- M-theory on $CY_3 \times S^1/\mathbb{Z}_2$ or on $\mathcal{M}^7$ with G_2 holonomy

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$\Rightarrow$ Calabi-Yau threefold CY_3 or fourfold CY_4.

Calabi-Yau Space

Q: What is Calabi-Yau?

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Calabi-Yau n-folds is a complex n-dimentional compacted Kähler Manifold satisfied:

- Its first chern class vanish, i.e $c_1(M) = 0 \in H^2(M, \mathbb{Z})$.
- The normal bundle $K_M = \wedge^n T^*(1,0)(M)$ is trivial since $c_1(K_M) = -c_1(M)$
- There exist a unique no where vanishing holomophic n-form, $\Omega_n \in \Omega^{n,0}(M)$, $d\Omega_n = 0$
- The Ricci tensor vanish, i.e. $R_{mn} = 0$
- The holonomy group of M is $SU(n)$

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- Many string phenomenology is built in Type IIB Calabi-Yau orientifold with $O3/O7$-plane.

$$\mathcal{O}=\begin{cases}\Omega_p\,\sigma & \text{with} \quad \sigma^*(J)=J, \quad \sigma^*(\Omega_3)=\Omega_3\,, \quad O5/O9 \\ (-)^{F_L}\,\Omega_p\,\sigma & \text{with} \quad \sigma^*(J)=J, \quad \sigma^*(\Omega_3)=-\Omega_3, \quad O3/O7\end{cases}$$

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- In Type IIB orientifold, Complex,dilaton moduli decoupled with Kähler moduli.
  - Complex and dilaton moduli can be stabilized by background fluxes at tree level. Gukov/Vafa/Witten
  - Kähler moduli can be stabilized by non-perturbative effects (KKLT, Large Volume scenario). Kachru/Kallosh/Linde/Trivedi, Balasubramanian/Berglund/Colon/Quevedo

How to Construct Calabi-Yau Database

- Toric Calabi-Yau Borisov, Batyrev, Cox, Kreuzer, Skarke ......
  - Hypersurface $\hookrightarrow$ 473,800,776 reflexive polyhedra in 4D
    Kreuzer,Skarke,Altman,Gray,He,Jejjala,Nelson,...
  - Hypersurface $\hookrightarrow$ weighted project space Kreuzer,Skarke,...
- Complete Intersection Calabi-Yau (CICY)
  - Complete intersection hypersurfaces $\hookrightarrow$ Product of projective spaces
    7,890 configuration matrices for CY3
    Hubsch,Candelas,Dale,Lutaken,Schimmrigk,Green,...
    921,497 configuration matrices for CY4 Gray,Haupt,Lukas,...
  - Generalized Complete Intersection Calabi-Yau Manifolds (gCICY)
    Anderson, Apruzzi, XG, Gray, Lee, 15'

Calabi-Yau 3-folds Database

- CICY (# 7890), gCICY ($\# > \mathcal{O}(10^3)$) and toric CY ($\# > \mathcal{O}(10^{10})$).
  Candelas/Dale/Lutken/Schimmrigk, Anderson/XG/Gray/Lee, Anderson/Apruzzi/XG/Gray/Lee, Kreuzer/Skarke, Altman/Gray/He/Jejjala/Nelson

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- Orientifold involution

$$\sigma = \begin{cases} \text{Reflection} : \{ x_i \leftrightarrow -x_i, \cdots \} & h^{1,1}_{-}(X) = 0 \\ \text{Exchange involution} : \{ x_i \leftrightarrow x_j, \cdots \} & h^{1,1}_{-}(X) \neq 0 \end{cases}$$

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$h^{1,1}_-(X) \neq 0$ is important to solve the chirality issue for global model building (Combine partical physics and moduli stabilization and inflation in a single set-up). Blumenhagen/Moster/Plauschinn, Cicoli/Mayrhofer/Valandro/Quevedo/Krippendorf, Balasubramanian/Berglund/Braun/Garcia-Etxebarria, Grimm/Weigand/Kerstan $\cdots$

  - D-brane at singularity
  - Fluxed Instanton

Searching and Classification of Orientifold CY3s

- Based in the favorable CICY database Anderson/XG/Gray/Lee, orientifold CICYs has been studied recently. Carta/Moritz/Westphal

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  Altman/Carifio/ XG /Nelson

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  Altman/Carifio/ XG /Nelson
- Among total 646903 CYs with $h^{1,1}(X) \leq 6$, only 5% of them admits a proper divisor exchange orientifold.
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- Suitable for Machine Learning to extend our result to higher $h^{1,1}$ to search and classify orientifold CYs. XG /Zhou
- Based on our works, some new progress is under going. Crino/Quevedo/ Schachner/Valandro

Outline

Toric Variety

Definition: Set $d, m \in \mathbb{N}$ and $n = d + m$, a $d-$dim Toric Variety Δ° is defined by the coset space of homogenous coordinates $x_i,\ i = 1, \dots n$:

$$\Delta^\circ = \frac{\mathbb{C}^n - Z}{(\mathbb{C}^*)^m}$$

where Z is the zero-set. $(\mathbb{C}^*)^m$ defines an equivelant relation through the charge matrix Q_i^1:

$$(x_1, \dots, x_n) \sim (\lambda^{Q_1^a} x_1, \dots, \lambda^{Q_n^a} x_n) \quad \forall a = 1, \dots, m, \quad \forall \lambda \in \mathbb{C}^*,$$

Definition: Let Z be the zero set of a toric variety Δ°. The Stanley-Reisner ideal I is the minimal ideal containing square free monomials corresponding to the different subsets of the zero set:

$$I_{\Delta^\circ} = \left\{ \prod_{j=1}^{k} x_{i_j} \quad | \quad \{x_{i_j} = 0\} \in Z_i, \quad i = 1, \dots, n. \right\}.$$

We use $(\Delta^\circ \equiv \bigtriangledown, N)$ to denote the Toric variety. In PALP and SAGE, they call Δ° dual-Polytopy. In fact, we can have two ways to define the toric variety:

- coordinates x_i in charge matrix Q. $\Leftrightarrow$ vertex in Dual-Lattice Polytopy Δ°.

Triangulations

Usually, a toric variety has some singularity, need to be resolved.

- Replace the isolated singularity by higher dimension curve is called Resolution of singularity. Using $\mathbb{P}^n$ to do it is called Blow-up. For a given singularity there may exist several resolution ways. The different ways may be connected by so-called Flop Transition.
- There are other methods to resolve the singularity such as deformation.
- So we can see that resulotion is associated to decomposite the varity to some smaller pieces. This is called Triangulations or Simplicial Decomposition. A Maximal Triangulation means there is no further triangulations which can get more cone.
- Usually, we call the dual-Polytopy $\Delta^\circ \equiv (\bigtriangledown, N)$ together with their triangulation information the Ambient Space $\mathcal{A}$.

Toric Calabi-Yau Hypersurface

THM: If a toric variety is Calabi-Yau manifold, it is non-compact.

THM: Set Toric Variety Δ° defined by $x_1, \ldots, x_n$ and the associated GLSM charge $Q_1^a, \ldots, Q_n^a$. Set $G_1, \ldots, G_c$ are the homogenous polynomial in Δ°, then the complete intersection hypersurfaces $\mathcal{S}$ is given by:

$$\mathcal{S} = \{G_1 = 0\} \cap \cdots \cap \{G_c = 0\}.$$

Furthermore,

$$\mathcal{S} \text{ is Compact Calabi} - \text{Yau} \Leftrightarrow ||G_1||^a + \cdots + ||G_c||^a = Q_1^a + \cdots + Q_n^a, \quad \forall a.$$

i.e, the sum of the degree of homogenous polynormail is equal to the sum of charge of homogenous coordinates in Δ°.

Polynomial Representations of Toric CYs

Definition: The Polytope (Δ, M) dual of $\Delta^\circ = (\bigtriangledown, N)$ is defined by:

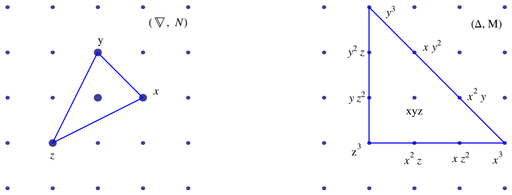
$$\Delta = \{m \in M_{\mathbb{R}} = \mathbb{Z}^d \quad | \quad \langle v_i | m \rangle \geq -1,\ \forall v_i \in \bigtriangledown\} \subset M_{\mathbb{R}}.$$

(Δ, M) is also called the Newton polytope. If Δ is still an integer lattice polytopy, we call it Reflexive.

For (Δ, M), the compact smooth Calabi-Yau anticanonical hypersurface can be defined by the vanishing of a homogeneous polynomial $\{P = 0\}$.

$$P = \sum_{m \in \Delta} a_m M_m = 0, \qquad \text{where} \qquad M_m = \prod_{i=1}^{d} x_i^{\langle m, v_i \rangle + 1}.$$

Example:. The dual-polytopy and polytopy of $\mathbb{P}^2$ and its polynomial representation:



Outline

Polytopes, Triangulations and Geometries

- Favorable Description: When Toric divisor classes on the Calabi-Yau hypersurface X are all descended from ambient space $\mathcal{A}$.

$$h^{1,1}(X) = \dim(H^{1,1}(X)) \cong \dim(\mathrm{Pic}(\mathcal{A})) = h^{1,1}(\mathcal{A})$$

- MPCP: Maximal Projective Crepant Partial (MPCP) desingularization involves the triangulation of the polar dual reflexive polytope Δ^*, which contains at least one fine, star, regular triangulation (FSRT).
- Wall's theorem: The compact Calabi-Yau 3-folds are classified by the Hodge numbers, the intersection numbers, and the second Chern Class.

$\Longrightarrow$ Geometry-wise description: Glue together the various phases of the complete Kähler cone corresponding to a distinct Calabi–Yau threefold geometry.

Proper Involution σ: NID

$$\sigma : x_i \leftrightarrow x_j \quad \Longrightarrow \quad \sigma^* : D_i \leftrightarrow D_j.$$

- In favorable case, restricts strightforward to the Calabi-Yau hypersurface.
- $D_\pm = D_i \pm D_j \in H^{1,1}_\pm(X/\sigma^*)$

Proper Involutions:

- Non-Trivial Identity Divisor: $H^\bullet(D_i) \cong H^\bullet(D_j)$ with different wights $\mathcal{O}(D)$.

  - Completely Rigid Divisors: $h^\bullet(D) = \{h^{0,0}(D), h^{0,1}(D), h^{0,2}(D), h^{1,1}(D)\} = \{1, 0, 0, h^{1,1}(D)\}$.
  - Wilson Divisors: $h^\bullet(W) = \{1, h^{1,0}, 0, h^{1,1}\}$. $h^{1,0}_+ = 1$ characterize the zero modes of poly-instanton, which can not be lifted by background fluxes.
  - Deformation divisors such as $K3$.

Proper Involution σ: Consistent

- Symmetry of Stanley-Reisner Ideal $\mathcal{I}_{SR}(\mathcal{A})$: To ensure the involution to be an automorphism of $\mathcal{A}$, leaving invariant the exceptional divisors from resolved singularities.
- Symmetry of the linear ideal $\mathcal{I}_{lin}(\mathcal{A})$: To ensures the defining polynomial of CY remains homogeneous under involution.

$$A^{\bullet}(\mathcal{A}) \cong \frac{\mathbb{Z}(D_1, \cdots, D_k)}{\mathcal{I}_{lin}(\mathcal{A}) + \mathcal{I}_{SR}(\mathcal{A})} .$$

Due to the favorability condition on the Calabi-Yau threefold hypersurface we have

$$A^1(\mathcal{A}) \cong H^{1,1}(\mathcal{A}) \cong \mathrm{Pic}(\mathcal{A}) \cong \mathrm{Pic}(X) \cong H^{1,1}(X) \cong A^1(X) ,$$

thus the toric triple intersection tensor defined in the Chow ring of X.

$$d_{ijk} = \int_X D_i \wedge D_j \wedge D_k \equiv D_i \cdot D_j \cdot D_k \cdot X \quad \text{and} \quad X = -K_{\mathcal{A}} = \sum_{i=1}^{k} D_i$$

$\Longrightarrow$ Triple intersection tensor is invariant under involution σ.

Types of Proper Involution σ

- Triangulation-wise proper involution: The involutions present at the triangulation level - that is, within a single chamber of the Kähler cone of a given geometry.
- Geometry-wise proper involution: The involutions which are globally consistent across all disjoint phases of the Kähler cone for each unique Calabi-Yau geometry.
- Each of the geometry-wise proper involutions may correspond to several triangulation-wise involutions which can span an entire CY geometry.

Fixed Orientifold Planes I : Invariant CY Hypersurface Polynomial

$$\mathcal{A} = \frac{\mathbb{C}^k \setminus Z}{(\mathbb{C}^*)^{k-4} \times G},$$

The geometry can be described by $\{x_1, ..., x_k\}$ and their $\mathbb{C}^*$ equivalence classes

$$(x_1, ..., x_k) \sim (\lambda^{\mathbf{W}_{i1}} x_1, ..., \lambda^{\mathbf{W}_{ik}} x_k),$$

$$P = \sum_{m \in \Delta} a_m M_m = 0, \qquad \text{where} \qquad M_m = \prod_{i=1}^{k} x_i^{\langle m, n_i \rangle + 1}.$$

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Define the set of monomials $\mathcal{M} = \{M_m | m \in \Delta\}$. Then for $M_m, M_{m'} \in \mathcal{M}$, we identify three cases:

1. $\sigma(M_m) = M_m \quad \Rightarrow \qquad a_m$ is generic,
2. $\sigma(M_m) = M_{m'}, \quad m \neq m' \quad \Rightarrow \qquad a_m = a_{m'}$,
3. $\sigma(M_m) \notin \mathcal{M} \quad \Rightarrow \qquad a_m = 0$.

$\Longrightarrow P \mapsto P_{symm}$, such that $\sigma(P_{symm}) = P_{symm}$ in addition with $\sigma^* J = J$.

Fixed Orientifold Planes II : Minimal Generators $\mathcal{G}$

$\mathcal{G}$: generated by homogeneous polynomials $y(x_1, ..., x_k)$ that are (anti-)invariant under σ.

$$\mathcal{G} = \mathcal{G}_0 \cup \mathcal{G}_+ \cup \mathcal{G}_- \,.$$

- The unexchanged coordinates in $\mathcal{G}_0$ are known from our choice of involution.
- To finding the non-trivial even and odd parity generators in $\mathcal{G}_+$ and $\mathcal{G}_-$, we must consider all possible non-trivial "sub-involutions" given by the non-empty subsets of $\{\sigma_1, ..., \sigma_n\} \subseteq \sigma$ of size $1 \leq m \leq n$.

$$y_\pm(\mathbf{a}) = x_{i_1}^{a1} x_{i_2}^{a2} \cdots x_{i_m}^{am} \pm x_{j_1}^{a1} x_{j_2}^{a2} \cdots x_{j_m}^{am} \,,$$

The condition for homogeneity, in terms of the columns $\mathbf{w}_{i_s}$ and $\mathbf{w}_{j_s}$ of the weight matrix $\mathbf{W}$ is given by:

$$a_1(\mathbf{w}_{i_1} - \mathbf{w}_{j_1}) + a_2(\mathbf{w}_{i_2} - \mathbf{w}_{j_2}) + \cdots + a_m(\mathbf{w}_{i_m} - \mathbf{w}_{j_m}) = 0 \,.$$

Fixed Orientifold Planes III : Naive Fixed Point Loci

- Segre embedding: $\{x_1, ..., x_k\} \mapsto \{y_1, ..., y_{k'}\} \equiv \mathcal{G}$ and construct a new weight matrix $\tilde{\mathbf{W}}$ for $\{y_i\}$.
- The exchange involution has transfomed into reflection.
- Point-wise fixed point for codim-1 divisor: $\sigma : y \mapsto -y$, so that $D = \{y = 0\}$ is fixed.
- Point-wise fixed point for codim larger than one: check whether the involution forces a subset of generators $\mathcal{F} \subseteq \mathcal{G}$ to vanish simultaneously. In fact, check $\mathcal{F} \cap \mathcal{G}_- \neq \emptyset$.
    - Redundancy: The torus $\mathbb{C}^*$ actions provide $r = \text{rank}(\tilde{\mathbf{W}})$ additional degrees of freedom for the generators to avoid being forced to zero.
    - In each subset of generators $\mathcal{F}$, we check for this by solving the system of equations
    $$\lambda_1^{\tilde{W}_{1i}} \lambda_2^{\tilde{W}_{2i}} \cdots \lambda_r^{\tilde{W}_{ri}} = \sigma(y_i)/y_i, \quad i = 1, ..., k' .$$
    By the construction of the generator y_i, the right-hand side is equal to ± 1. The set is point-wise fixed if this equation is solvable in the λ_i.

Fixed Orientifold Planes IV : $\mathcal{I}_{SR}$ and CY Transversality

- Check whether each point-wise fixed loci lie in Stanley-Reisner ideal $\mathcal{I}_{SR}$.
- The definition of $\mathcal{I}_{SR}$ leads $\mathcal{A}$ to be spllited into different patches: U_i.
- For a given fixed set $\mathcal{F} \equiv \{y_1, \cdots, y_p\}$, we compute in each sector U_i the dimension of the ideal generated by

$$\mathcal{I}_{ip}^{fixed} = \langle U_i, P_{symm}, y_1, ..., y_p \rangle \, .$$

  If dim $\mathcal{I}_{ip}^{fixed} < 0$ for all U_i, then $\mathcal{F}$ does not intersect the X.
- For each subset that is not discarded, we repeat this calculation for the ideal with one fixed set generator dim $\mathcal{I}_{i1}^{fixed}$, and then two dim $\mathcal{I}_{i2}^{fixed}$, etc. until dim $\mathcal{I}_{i\ell}^{fixed}$ = dim $\mathcal{I}_{ip}^{fixed}$ when adding more generators to the ideal no longer changes the dimension for any region U_i. Then, the intersection $\{y_1 = \cdots = y_\ell = 0\}$ of these generators gives the final point-wise fixed locus, with redundancies eliminated.
- An O3 plane corresponds to a codimension-3 point-wise fixed subvariey, an O5 plane has codimension-2, an O7 plane has codimension-1.

Fixed Orientifold Planes V : Smoothness

- Check whether the invariant Calabi-Yau hypersurface defined by P_{symm} is smooth. This is important to determine whether an involution is a free action. We do this by checking by setting up the ideals

$$\mathcal{I}_i^{smooth} = \langle U_i,\ P_{symm},\ \frac{\partial P_{symm}}{\partial x_1},\ ...,\ \frac{\partial P_{symm}}{\partial x_k} \rangle.$$

  for each region U_i allowed by the $\mathcal{I}_{SR}$, and computing the dimension. If dim $\mathcal{I}_i^{smooth} < 0$ for all U_i, then the invariant Calabi-Yau hypersurface is smooth.
- If no O-planes exist and the invariant Calabi-Yau hypersurface is smooth, then the involution defines a $\mathbb{Z}_2$ free action on X.

Tadpole Cancelation and String Vacua

- Cancel the $D7$-brane tadpole by simply placing eight $D7$-branes on top of the $O7$-plane.
- $D3$-brane tadpole condition simplified to:

$$N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{4} \equiv -\, Q_{D3}^{loc}.$$

with $N_{\text{flux}} = \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3$, $N_{\text{gauge}} = -\sum_a \frac{1}{8\pi^2} \int_{D_a} \text{tr}\mathcal{F}_a^2$, and N_{D3}, N_{O3} the number of $D3$-branes, $O3$-planes respectively.

- String Vacua: The $D3$-tadpole cancelation condition requires the total D3-brane charge Q_{D3}^{loc} of the seven-brane stacks and $O3$-planes to be an integer. If the involution passes this naive tadpole cancellation check, we will denote our geometry as a "naive orientifold Type IIB string vacua".

Hodge Number Splitting

- Under the involution, the dimensions of Hodge numbers split

$$H^{p,q}(X/\sigma^*) = H^{p,q}_+(X/\sigma^*) \oplus H^{p,q}_-(X/\sigma^*)\,.$$

- Example: $h^{1,1}(X) = 3$, admitting a proper orientifold involution $\sigma^* : D_2 \leftrightarrow D_3$. Suppose the divisor classes $\{D_1, D_2, D_3\}$ form a basis for $H^{1,1}(X;\mathbb{Z})$. Then, the Kähler form can be expanded as

$$J = t_1 J_1 + t_2 J_2 + t_3 J_3 = t_1 D_1 + t_2 D_2 + t_3 D_3\,,$$

  with $t_1, t_2, t_3 \in \mathbb{Z}$.

$$J = \sigma^* J = t_1 D_1 + t_2 D_3 + t_3 D_2 = t_1 J_1 + t_3 J_2 + t_2 J_3\,.$$

  Then we note that we must have $t_2 = t_3 = t_+$, for some $t_+ \in \mathbb{Z}$. Defining the even and odd parity eigendivisors $D_\pm = D_2 \pm D_3$, we can write

$$J = t_1 D_1 + t_+ D_+\,.$$

  so $h^{1,1}_+(X/\sigma^*) = 2$ and $h^{1,1}_-(X/\sigma^*) = 1$

- We can count the new $h^{2,1}(X/\sigma^*)$ if it is smooth.

Outline

Example: $h^{1,1}(X) = 4, h^{2,1}(X) = 64.$

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
0	0	0	1	0	1	0	0
0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1
1	0	0	1	0	0	1	1

.

- $\mathcal{I}_{SR} = \langle x_1 x_8,\ x_3 x_7,\ x_4 x_6,\ x_1 x_4 x_7,\ x_2 x_3 x_5,\ x_2 x_5 x_6,\ x_2 x_5 x_8 \rangle$
- The linear ideal, which fixes toric divisor redundancies, is given by

$$\mathcal{I}_{lin} = \begin{array}{cccccccccccccccccc} \langle & -D_1 & - & D_2 & - & D_3 & - & D_4 & + & 0 & + & D_6 & + & D_7 & + & D_8, & \\ + & 0 & + & 0 & + & D_3 & + & D_4 & + & 0 & - & D_6 & - & D_7 & & 0, & \\ - & D_1 & & 0 & - & D_3 & - & D_4 & - & D_5 & + & D_6 & + & D_7 & + & D_8, & \\ + & 0 & + & 0 & + & 0 & + & D_4 & + & D_5 & - & D_6 & + & 0 & - & D_8 & \rangle, \end{array}$$

and a basis in $H^{1,1}(X;\mathbb{Z})$ given by $J_1 = D_1,\ J_2 = D_2,\ J_3 = D_3,\ J_4 = D_6$.

$$h^\bullet(D_1) = \{1,0,0,9\}, \quad h^\bullet(D_2) = h^\bullet(D_4) = h^\bullet(D_5) = h^\bullet(D_7) = \{1,0,1,21\}$$
$$h^\bullet(D_3) = h^\bullet(D_6) = \{1,0,0,12\}, \quad h^\bullet(D_8) = \{1,0,2,30\}$$

- Only geometry-wise proper involution: $\sigma : x_3 \leftrightarrow x_6, \quad x_4 \leftrightarrow x_7$
- $\sigma^* \Omega_3 = -\Omega_3$. One would expect $O3/O7$-system.

Orientifold Planes I : Minimal Generators $\mathcal{G}$

- $\mathcal{G}_0 = \{x_1, x_2, x_5, x_8\}$.
- $\sigma_1 : \mathbf{x_3} \leftrightarrow \mathbf{x_6} \Rightarrow \quad \mathcal{G}_+ = \{x_3 x_6\}, \;\; \mathcal{G}_- = \emptyset$
- $\sigma_2 : \mathbf{x_4} \leftrightarrow \mathbf{x_7} \Rightarrow \quad \mathcal{G}_+ = \{x_4 x_7\}, \;\; \mathcal{G}_- = \emptyset$
- $\sigma : \mathbf{x_3} \leftrightarrow \mathbf{x_6}, \;\; \mathbf{x_4} \leftrightarrow \mathbf{x_7}$: $x_3^m x_4^n \pm x_6^m x_7^n$ for $m, n \in \mathbb{Z}$.
  The homogeneity of this binomial is determined by the following condition on the weights

  $$m(\mathbf{W}_{i3} - \mathbf{W}_{i4}) + n(\mathbf{W}_{i6} - \mathbf{W}_{i7}) = \mathbf{0}\,.$$

  The kernel is generated by the vector $(m, n) = (1, 1)$, so
  $\mathcal{G}_+ = \{x_3 x_4 + x_6 x_7\}$ and $\mathcal{G}_- = \{x_3 x_4 - x_6 x_7\}$.
- Serge embbeding:

$$y_1 = x_1\,, \quad y_2 = x_2\,, \quad y_3 = x_5\,, \quad y_4 = x_8\,, \quad y_5 = x_3 x_6\,,$$
$$y_6 = x_4 x_7\,, \quad y_7 = x_3 x_4 + x_6 x_7\,, \quad y_8 = x_3 x_4 - x_6 x_7\,.$$

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	
0	0	0	0	1	1	1	1	λ_1
0	1	1	1	0	0	0	0	λ_2
1	0	0	1	0	2	1	1	λ_3

Orientifold Planes II: Naive Fixed Loci

- $y_8 \mapsto -y_8$: $F_1 = \{y_8 = 0\}$ is a point-wise fixed, codimension-1 subvariety.
- Check whether any subset $\mathcal{F} \equiv \{y_1, \cdots, y_p\}$ of the generators can neutralize the odd parity of y_8, becoming fixed themselves in the process.
- We begin our scan with the largest set of generators and work our way down. The largest set we can choose has 4 generators, since their simultaneous vanishing defines a set of isolated points on $\mathcal{A}$.

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- We begin our scan with the largest set of generators and work our way down. The largest set we can choose has 4 generators, since their simultaneous vanishing defines a set of isolated points on $\mathcal{A}$.
- Consider $F_2 = \{y_1 = y_2 = y_3 = y_7 = 0\}$ to be fixed, we must use the three independent $\mathbb{C}^*$ actions to neutralize the odd parity of y_8 while leaving everything else invariant.

$$(y_4, y_5, y_6, -y_8) \sim (\lambda_2\lambda_3 y_4, \lambda_1 y_5, \lambda_1\lambda_3^2 y_6, \lambda_1\lambda_3 y_8) = (y_4, y_5, y_6, y_8)$$

where $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}^*$.

$$\lambda_2\lambda_3 = 1 \quad \lambda_1 = 1 \quad \lambda_1\lambda_3^2 = 1. \quad \lambda_1\lambda_3 = -1\,.$$

$\Longrightarrow (\lambda_1, \lambda_2, \lambda_3) = (1, -1, -1)$ and so F_2 is indeed a point-wise fixed set.

Orientifold Planes III: True Loci & String Vacua

- The fixed point set $F_2 = \{y_1 = y_2 = y_3 = y_7 = 0\}$ can be written in terms of the original coordinates $\{x_1 = x_2 = x_5 = 0\} \cap \{x_3 x_4 = -x_6 x_7\}$. Substitutions in P_{symm}:

$$P_{symm} = a_{48}(x_3^2 x_4 x_6 x_8^3 + x_3 x_6^2 x_7 x_8^3) = a_{48} x_3 x_6 x_8^3 y_7 \,.$$

  - $x_2 x_3 x_5 \in \mathcal{I}_{SR} \implies x_3 \neq 0, \qquad x_2 x_5 x_6 \in \mathcal{I}_{SR} \implies x_6 \neq 0,$
    $x_2 x_5 x_8 \in \mathcal{I}_{SR} \implies x_8 \neq 0$

  $\implies y_7 = 0$ for P_{symm} vanishing, which is a redundancy.

$$F_2' = \{y_1 = y_2 = y_3 = 0\}$$

- There are 17 U_i, by checking F_1 and F_2' as

$$\mathcal{I}_{ij}^{fixed} = \langle U_i,\ P_{symm},\ F_j \rangle$$

  we can determine F_1 is an O7 plane, while F_2' is an O3 plane locus.

- In fact, there are only one O7 and one $O3$-plane, and we have:

$$N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{4} = \frac{1+39}{4} = 10\,.$$

  Geometry-wise "naive orientifold type IIB string vacua".

Hodge Number Splitting

- Holomorphicity condition $\Longrightarrow H^{p,q}(X/\sigma^*) = H^{p,q}_{+}(X/\sigma^*) \oplus H^{p,q}_{-}(X/\sigma^*)$
- Favrability $\Longrightarrow H^{1,1}(\mathcal{A}) \cong \mathrm{Pic}(\mathcal{A}) \cong \mathrm{Pic}(X) \cong H^{1,1}(X)$
  We can always expand the Kähler form in terms of the divisor classes.

$$J = t_1 J_1 + t_2 J_2 + t_3 J_3 + t_4 J_4 = t_1 D_5 + t_2 D_6 + t_3 D_7 + t_4 D_8$$

The Kähler form must be invariant under the pullback of involution,

$$J = \sigma^* J = t_1 D_5 + t_2 D_3 + t_3 D_4 + t_4 D_8 = t_1 J_1 + t_2 D_3 + t_3 D_4 + t_4 J_4 \qquad (1)$$

$$\Longrightarrow D_3 = J_1 + J_3 - J_4 \qquad \text{and} \qquad D_4 = -J_1 + J_2 + J_4 \, ..$$

$$t_1 + t_2 - t_3 = t_1 , \quad t_3 = t_2 , \quad t_2 = t_3 , \quad -t_2 + t_3 + t_4 = t_4 \, .$$

$$h^{1,1}_{+}(X/\sigma^*) = 3, \qquad h^{1,1}_{-}(X/\sigma^*) = 1$$

- The result is basis independent.

Outline

$h^{1,1}(X)$	1	2	3	4	5	6	Total
# of Favorable Polytopes	5	36	243	1185	4897	16608	22974
# of Favorable Triangulations	5	48	525	5330	56714	584281	646903
# of Favorable Geometries	5	39	305	2000	13494	84525	100368
% of Favorable Triangulations Scanned	80	100	99.8	99.66	99.41	99.01	99.01

Table 1: The favorable polytopes, triangulations, geometries for $h^{1,1}(X) \leq 6$.

$h^{1,1}(X)$	1	2	3	4	5	6	Total
Triangulation-wise proper NID exchange involutions							
# of Polytopes contains Involutions	0	1	25	166	712	2172	3076
# of Geometries contains Involutions	0	1	26	273	1559	6590	8449
# of Triangulations contains Involutions	0	1	31	405	3372	21566	25375
# of Involutions	0	6	51	516	4085	23805	28463
Geometry-wise proper NID exchange involutions							
# of Polytope contains Involutions	0	1	16	96	330	958	1401
# of Geometries contains Involutions	0	1	17	183	911	3370	4482
# of Involutions	0	6	28	259	1219	4148	5660
% of Polytope contains Involutions	0	2.78	6.58	8.10	6.74	5.77	6.10
% of Geometries contains Involutions	0	2.56	5.57	9.15	6.75	3.99	4.47

Table 2: Statistic counting on the triangulation/geometry-wide Non-trivial Identical Divisors exchange involutions in favorable polytopes, triangulations and geometries.

Number of pairs of Non-trivial Identical Divisors (NID) under involutions							
$h^{1,1}(X)$	1	2	3	4	5	6	Total
Triangulation-wise proper Involutions							
# of Involutions	0	6	51	516	4085	23805	28463
del Pezzo surface dP_n, $n \leq 8$	0	0	12	238	2233	14507	17090
Rigid surface dP_n, $n > 8$	0	0	14	512	5659	32481	38666
(exact-)Wilson surface	0 (0)	0 (0)	5 (0)	40 (5)	177 (80)	744 (411)	966 (496)
K3 surface	0	0	65	300	619	1976	2960
SD1 surface	0	0	9	47	418	2190	2664
SD2 surface	0	18	8	33	109	459	627
del Pezzo and K3	0	0	0	9	98	572	679
del Pezzo and (Exact-)Wilson	0 (0)	0 (0)	1 (0)	28 (0)	95 (9)	667 (286)	791 (295)
K3 and (Exact-)Wilson	0 (0)	0 (0)	8 (0)	12 (4)	43 (7)	101 (9)	156 (20)
del Pezzo, K3 and (Exact-)Wilson	0 (0)	0 (0)	0 (0)	0 (0)	28 (0)	87 (2)	115 (2)
Geometry-wise proper Involutions							
# of Involutions	0	6	28	259	1219	4148	5660
del Pezzo surface dP_n, $n \leq 8$	0	0	8	107	634	2660	3409
Rigid surface dP_n, $n > 8$	0	0	8	259	1973	6198	8438
(Exact-)Wilson surface	0 (0)	0 (0)	5 (0)	28 (2)	48 (4)	136 (75)	217 (81)
K3 surface	0	0	28	215	219	527	989
SD1 surface	0	0	8	23	102	216	349
SD2 surface	0	18	6	18	39	84	165
del Pezzo and K3	0	0	0	0	26	156	182
del Pezzo and (Exact-)Wilson	0 (0)	0 (0)	1 (0)	19 (0)	40 (1)	109 (40)	169 (41)
K3 and (Exact-)Wilson	0 (0)	0 (0)	8 (0)	12 (4)	13 (4)	23 (3)	56 (11)
del Pezzo, K3 and (Exact-)Wilson	0 (0)	0 (0)	0 (0)	0 (0)	4 (0)	16 (2)	20 (2)

Classification of O-plane fixed point locus							
$\mathbf{h^{1,1}(X)}$	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Triangulation-wise proper Involutions							
# of Involutions	0	6	51	516	4085	23772	28430
O3	0	0	9	253	2640	18193	21083
O5	0	6	20	157	1006	3279	4468
O7	0	0	31	328	3005	20137	23501
O3 and O7	0	0	9	222	2566	17826	20623
Free Action	0	0	0	0	0	1	1
Geometry-wise proper Involutions							
# of Involutions	0	6	28	259	1219	4148	5660
O3	0	0	4	82	557	2611	3254
O5	0	6	16	106	488	929	1545
O7	0	0	12	124	691	3082	3909
O3 and O7	0	0	4	53	523	2475	3055
Free Action	0	0	0	0	0	1	1

Table 4: Classification of O-plane fixed point locus and free actions under the triangulation/geometry-wise proper involutions.

Naive Orientifold Type IIB String Vacua with $O3/O7$-system							
$\mathbf{h^{1,1}(X)}$	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Triangulation-wise proper Involutions							
# of Involutions	0	6	51	516	4085	23772	28430
Contains O3 & O7	0	0	9	206	2346	15234	17795
Contains Only O3	0	0	0	31	74	355	460
Contains Only O7	0	0	22	102	386	1950	2460
Total String Vacua	0	0	31	339	2806	17539	20715
Geometry-wise proper Involutions							
# of Involutions	0	6	28	259	1219	4148	5660
Contains O3 & O7	0	0	4	48	455	1874	2381
Contains Only O3	0	0	0	29	34	136	199
Contains Only O7	0	0	8	68	149	529	754
Total String Vacua	0	0	12	145	638	2539	3334

Table 5: Classification of naive orientifold Type IIB string vacua under the triangulation/geometry-wise proper involutions.

Hodge number splitting								
$\mathbf{h^{1,1}(X)}$		**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Triangulation-wide proper Involutions								
# of Involutions		0	6	51	516	4085	23805	28463
# of $\mathbf{h^{1,1}_-}$	**1**	–	6	51	477	3682	20985	25201
	2	–	–	0	39	483	2618	3140
	3	–	–	–	0	0	202	202
	4	–	–	–	–	0	0	0
	5	–	–	–	–	–	0	0
Geometry-wide proper Involutions								
# of Involutions		0	6	28	259	1219	4148	5660
# of $\mathbf{h^{1,1}_-}$	**1**	–	6	28	277	1048	3413	4772
	2	–	–	0	32	171	661	864
	3	–	–	–	0	0	74	74
	4	–	–	–	–	0	0	0
	5	–	–	–	–	–	0	0

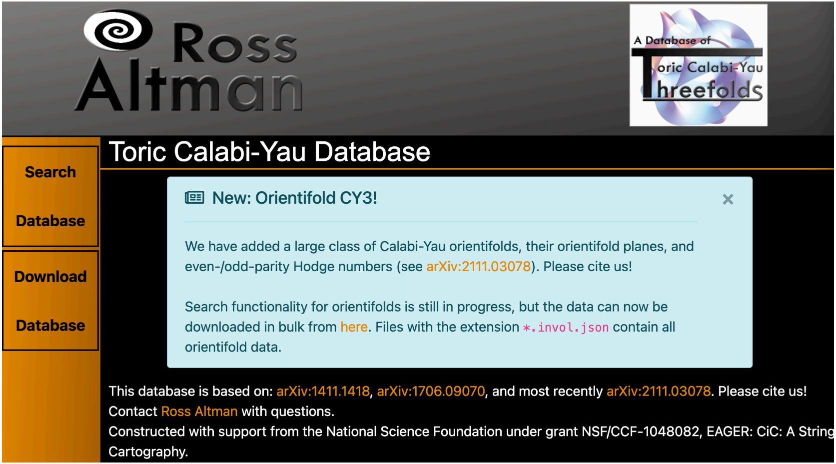
Table 6: Classification of $h^{1,1}(X/\sigma^*)$ splitting under the triangulation/geometry-wise proper involutions.

Outline

1. Calabi-Yau Manifolds
2. Toric Geometry
3. Orientifolds CYs
4. Example
5. Classifications
6. **Database**
7. Machine Learning

Database

http://www.rossealtman.com/toriccy



Example

- **Polytope #**: 566
- **Geometry #**: 1
- **Triangulation #**: 1
- **Involution #**: 1
- **h11**: 4
- **h21**: 64
- **Invol**: `{D3 -> D6,D6 -> D3,D4 -> D7,D7 -> D4}`
- **Geometry-wise Invol**: true
- **Volume Parity**: -1
- **# Sym CY Terms**: 48
- **Sym CY Poly**: In this case, all of the terms are symmetric, so this is the same as CY Poly. We avoid repeating it for brevity.
- **h11+**: 3
- **h11-**: 1
- **OPlanes**:

```
[
{ "OIDEAL" : [ "x3*x4-x6*x7" ], "ODIM" : 7 },
{ "OIDEAL" : [ "x1", "x2", "x5" ], "ODIM" : 3 }
]
```

- **Naive String Vacua**: True

Outline

Why ML?

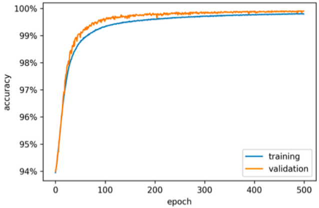
- Whether ML can pick out the orientifold property of a CYs.
- It was conjectured that the orientifold symmetry (at least the involution symmetry) on the CYs is already encoded in the polytope structure.
- Hard for higher $h^{1,1}$. Three difficulties.
- Rare Signal (around 5% for $h^{1,1} \leq 6$). It would be great even if we just train our machine to narrow down the candidate pool and increase the successful rate by one order.

Convolutional Neural Network (CNN)

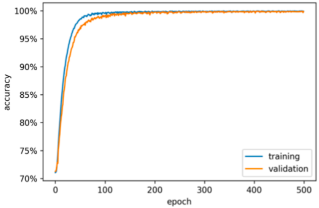
- Training data: 22960 polytopes, among them 1402 can result in an orientifold CYs and 996 can end up with a naive string vacua.
- Enlarge the data by 120 permutations: 2755200 training data.
- Layers (excluded the input layer):
  - one $2D$ convolution layer, with 25 filters, kernel size 3×3 and `ReLU` activation function,
  - one flatten layer, with default setup,
  - two full-connected layers (dense layers), both with 100 neurons and `ReLU` activation functions,
  - one dropout layer, with a dropout rate of 0.1,
  - one output layer (dense layer), with 2 neurons and `Softmax` activation function.
- Loss function: `Categorical Corssentropy`.
- Optimizer: `Adam`, with default learning rate.

Accuracy of classifier

Accuracy for unresolved data: 99.906% for orientifold & 99.802% for vacua.

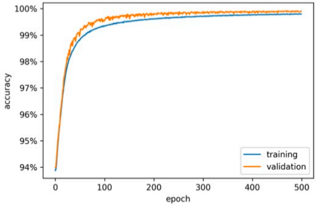


(a) Orientifold

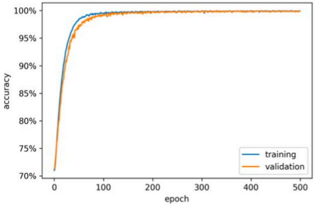


(b) Geometry-wise string vacua

Accuracy for resolved data: 99.907% for orientifold & 99.897% for vacua.



(a) Orientifold



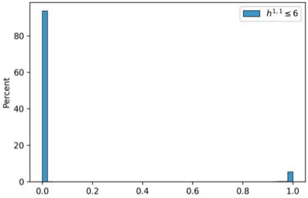
(b) Geometry-wise string vacua

Result of ML

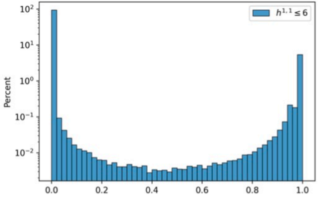
	Unresolved	Resolved
Orientifold	99.906%	99.907%
Naive Type IIB string vacua	99.802%	99.897%

Table 1: Test results for $h^{1,1} \leq 6$.

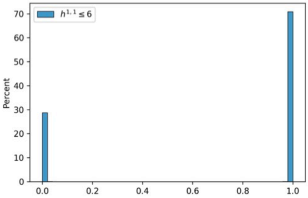
Probability histograms for training data



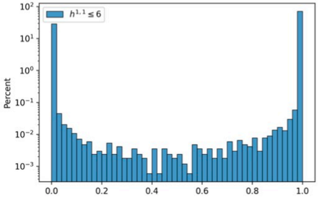
(a) Orientifold



(b) Orientifold (log-scale)



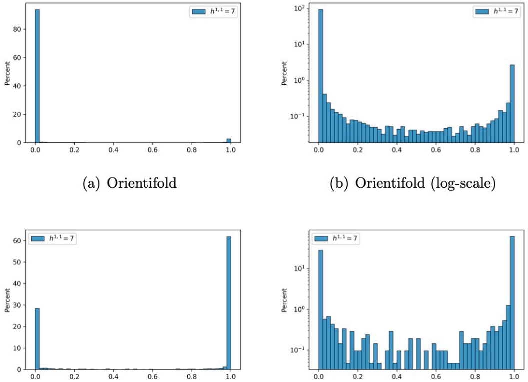
(c) Vacua



(d) Vacua (log-scale)

Prediction for higher $h^{1,1}$ ($h^{1,1} = 7$)

- Initial data: 50376 unresolved polytopes $\ll$ trained data (2755200)
- The trained model with parameters fixed.
- After classifier, among the polytopes with $h^{1,1} = 7$, 2086 of them may end up with orientifold CYs



(a) Orientifold (b) Orientifold (log-scale)

(c) Vacua (d) Vacua (log-scale)

Figure 5: Predicted probability histograms for data with $h^{1,1} = 7$.

Result of Trained Model

$\mathbf{h^{1,1}(X)}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**
# of Trianed Polytopes	5	36	243	1185	4897	16608	50376
# of "orientifold" Polytopes	0	1	16	96	330	958	2086
% of "orientifold" Polytopes	0	2.78	6.58	8.10	6.74	5.77	4.14

Table 2: Statistic counting on the polytopes which can result in orientifold Calabi-Yau. The result for $h^{1,1} \leq 6$ comes from [1] while for $h^{1,1} = 7$ comes from our trained neural network.

Example of prediction

- $h^{1,1}=7, h^{2,1}=53$, which was labeled as "orientifold " and "vacua".

$$\begin{matrix} 0 & 1 & -1 & -1 & -1 & 0 & 0 & -1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 & -1 & 0 & 1 & -1 & 0 & -1 & 1 \end{matrix}$$

- After explicitly triangulations, one can get the following topological data.

$$h^\bullet(D_1)=h^\bullet(D_2)=\{1,0,1,20\},\quad h^\bullet(D_{10})=h^\bullet(D_{11})=\{1,0,1,22\}$$
$$h^\bullet(D_4)=h^\bullet(D_8)=h^\bullet(D_9)=\{1,0,0,8\}$$

- It contains several possible involutions.
  If we choose: $\{D_1\leftrightarrow D_2, D_4\leftrightarrow D_9, D_{10}\leftrightarrow D_{11}\}$, we get four O7 plane with locus: $[D_4D_{10}-D_9D_{11}]$, $[D_3]$, $[D_5]$, $[D_6]$
  One can check it indeed satisfy the naive string vacua condition by:

$$\frac{36+9+7+12}{4}=16.$$

  If we choose another involution: $\{D_4\leftrightarrow D_9, D_{10}\leftrightarrow D_{11}\}$ we get four O7 plane and one O3 plane. However, it does not satisfy the naive string vacua condition.

Remarks for higher $h^{1,1}$

- Hard to check for higher $h^{1,1}$. Need to combine some other method of thiangulations. Demirtas/Long/McAllister/Stillman
- Favorable vs. Unfavorable Polytopes.
- Supervised training by generating enough initial orientifold CYs (we only need 30% of the data to train to get a high accuracy for $h^{1,1} \leq 6$). Use a subset of the database to learn something more complicated.

Ratio of Training Data	30%	20%	10%
Training Accuracy	99.70%	99.64%	99.22%
Validation Accuracy	99.75%	99.16%	91.90%
Test Accuracy	99.76%	99.14%	91.64%

- More complicated neural network may needed like Generative Adversarial Network (GAN) or Variational Autoencoder (VAE), which are in principle unsupervised training.

Conclusion

- Based on the favorable CY_3 constructed from Kreuzer-Skarke list, we push our upper bound to $h^{11} = 6$ by exact calculation.
- Instead of maximal triangulations, we consider all possible maximal projective crepant partial desingularizations (MPCP). The number of triangulations we analyzed increases from 2968 to 646903.

Conclusion

- Based on the favorable CY_3 constructed from Kreuzer-Skarke list, we push our upper bound to $h^{11} = 6$ by exact calculation.
- Instead of maximal triangulations, we consider all possible maximal projective crepant partial desingularizations (MPCP). The number of triangulations we analyzed increases from 2968 to 646903.
- We identify the topology of each divisors and determine the involutions which are globally consistent across all disjoint phases of the Kähler cone for each unique CY.
- Identify free action of involution and all possible fixed loci under non-trivial actions, thereby determining the type and location of O-planes.

Conclusion

- Based on the favorable CY_3 constructed from Kreuzer-Skarke list, we push our upper bound to $h^{11} = 6$ by exact calculation.
- Instead of maximal triangulations, we consider all possible maximal projective crepant partial desingularizations (MPCP). The number of triangulations we analyzed increases from 2968 to 646903.
- We identify the topology of each divisors and determine the involutions which are globally consistent across all disjoint phases of the Kähler cone for each unique CY.
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- The ML method gives a very high precision (99.96%) for identifying the polytopes which can result in an orientifold CY. This indicate the orientifold symmetry may encoded in the polytope structure itself.
- The ML method predict the polytopes which can result in an orientifold CY for higher h^{11}.

Thanks for your attention!