Fermionic BPS Wilson loops in four-dimensional $\mathcal{N} = 2$ superconformal gauge theories

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Based on 2205.01348 with Jun-Bao Wu

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In 4d $\mathcal{N}=4$ SYM the ordinary Wilson loop can be naturally generalized to the Maldacena-Wilson loop [Maldacena, 1998]

$$W[C] = \mathrm{Tr}\mathcal{P}\exp\left[-i\oint_C d\tau\left(A_\mu\dot{x}^\mu + \phi_I\theta^I|\dot{x}|\right)\right] \quad (1)$$

Origin: ordinary Wilson loop in 10d $\mathcal{N}=1$ SYM theory

Wilson loops in holography

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- Wilson loop $\leftrightarrow$ string in the bulk of AdS ending on the contour of the loop [Rey, Yee, 1998] [Maldacena, 1998]

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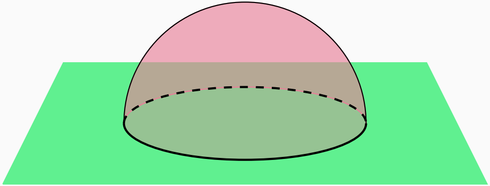
- Wilson loop ↔ string in the bulk of AdS ending on the contour of the loop [Rey, Yee, 1998] [Maldacena, 1998]
- Vacuum expectation value ↔ string worldsheet path integral

$$\langle W[C] \rangle = \mathrm{Tr} \int_{\partial X = C} \mathcal{D}X \exp(-\frac{\sqrt{\lambda}}{2\pi} S[X]) \xrightarrow{N,\lambda\to\infty} \exp(-\frac{\sqrt{\lambda}}{2\pi} A_{\mathrm{min}}).$$



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- Their expectation values can be computed exactly using supersymmetric localization technique [Pestun, 07].
- A famous example is the circular 1/2 BPS Wilson loop in $\mathcal{N} = 4$ SYM.

Localization

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- The expectation value of a circular BPS Wilson loop in $\mathcal{N} = 4$ SYM with gauge group $SU(N)$ can be reduced to computation to a Gaussian matrix model. [Erickson, Semenoff and Zarembo, 00] [Drukker and Gross, 00]

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$$\langle W\rangle = \frac{1}{Z}\int \mathcal{D}M \frac{1}{N}\mathrm{Tr}e^{M}\exp\left(-\frac{2N}{\lambda}\mathrm{Tr}M^2\right) = \frac{1}{N}L^1_{N-1}\left(-\frac{\lambda}{4N}\right)e^{\frac{\lambda}{8N}}$$

$$\xrightarrow{N\to\infty} \frac{2}{\sqrt{\lambda}}I_1(\sqrt{\lambda}) \xrightarrow{\lambda\to\infty} \sqrt{\frac{2}{\pi}}\lambda^{-\frac{3}{4}}e^{\sqrt{\lambda}}$$

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- $\sqrt{\lambda} = -\sqrt{\lambda}/(2\pi)A_{AdS_2}$. $A_{AdS_2} = -2\pi$ is the regularized area of AdS_2 minimal surface. [Berenstein, Corrado, Fischler, Maldacena, 98] [Drukker, Gross, Ooguri, 99]

- This can be derived by using the supersymmetric localization.

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- Fermionic Wilson loops: couples to bosonic and fermionic fields [Drukker, Trancanelli, 09].
- The motivation of our work is to explore similar fermionic BPS Wilson loops in four dimensions.

BPS Wilson loops in 3d superconformal Chern-Simons-matter theories

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- Supersymmetry transformations:

$$\delta A_\mu = \frac{1}{2}(\bar{\lambda}\gamma_\mu \epsilon + \bar{\epsilon}\gamma_\mu \lambda), \tag{2}$$

$$\delta \sigma = -\frac{i}{2}(\bar{\lambda}\epsilon + \bar{\epsilon}\lambda), \tag{3}$$

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- Bosonic BPS Wilson line on $x^\mu = \delta_0^\mu \tau$ [Gaiotto, Yin, 07]:

$$W_{\text{bos}} = \mathcal{P}\exp\left(-i\int d\tau L_{\text{bos}}\right), \quad L_{\text{bos}} = A_\mu \dot{x}^\mu + \sigma|\dot{x}|. \tag{4}$$

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- Bosonic BPS Wilson lines exist in theories with more supersymmetries. They always preserve two Poincaré and two conformal supercharges.

ABJM theory

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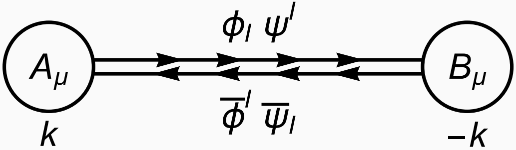
- It contains four complex scalars ϕ_I and four Dirac spinors ψ^I in bifundamental representation $(N, \bar{N})$.



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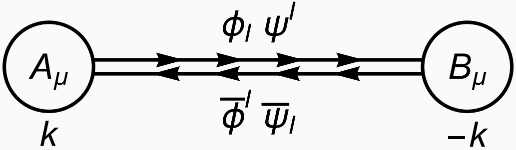

- It is holographically dual to M-theory on $\mathrm{AdS}_4\times \mathrm{S}^7/\mathbb{Z}_k$ or type IIA string theory on $\mathrm{AdS}_4\times \mathrm{CP}^3$.

Bosonic 1/6 BPS Wilson loops in ABJM theory

[Drukker, Plefka, Young 08] [Chen, Wu, 08] [Rey, Suyama, Yamaguchi, 08]

A 1/6 BPS Wilson line along $x^\mu = \tau\delta_0^\mu$ takes the form

$$W_{\text{bos}} = \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau \mathcal{A}_{\text{bos}}(\tau)\right), \tag{5}$$

$$\hat{W}_{\text{bos}} = \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau \hat{\mathcal{A}}_{\text{bos}}(\tau)\right), \tag{6}$$

$$\mathcal{A}_{\text{bos}} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} R^I{}_J \phi_I \bar{\phi}^J |\dot{x}|. \tag{7}$$

$$\hat{\mathcal{A}}_{\text{bos}} = B_\mu \dot{x}^\mu + \frac{2\pi}{k} S_I{}^J \bar{\phi}^I \phi_J |\dot{x}|, \tag{8}$$

$$R^I{}_J = S_I{}^J = \mathrm{diag}(-1,-1,1,1) \tag{9}$$

It is is basically the same as the bosonic BPS Wilson line in generic $\mathcal{N}=2$ Chern-Simons-matter theories.

1/2 BPS Wilson line in ABJM theory

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- However, the study of dual fundamental string solutions in $AdS_4 \times CP^3$ [Drukker, Plefka, Young, 08][Rey, Suyama, Yamaguchi, 08] indicates that there should be 1/2 BPS Wilson loops in ABJM theory.

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- Such Wilson loops were finally constructed in [Drukker, Trancanelli, 09] via including fermions

$$
\begin{aligned}
&W_{1/2} = \mathcal{P} e^{-i\int L_{1/2}}, \quad L_{1/2} = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix} \\
&\mathcal{A} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} M^I{}_J \phi_I \bar{\phi}^J |\dot{x}|, \quad \bar{f}_1 = \sqrt{\frac{2\pi}{k}} \bar{\zeta}_I \psi^I |\dot{x}|, \\
&\hat{\mathcal{A}} = B_\mu \dot{x}^\mu + \frac{2\pi}{k} N_I{}^J \bar{\phi}^I \phi_J |\dot{x}|, \quad f_2 = \sqrt{\frac{2\pi}{k}} \bar{\psi}_I \eta^I |\dot{x}|.
\end{aligned} \tag{10}
$$

Localization

- Evaluation of expectation values of bosonic BPS Wilson loops can be reduced to a matrix model, after using localization techniques. [Kapustin, Willett, Yaakov, 09]
- Expectation values depends on framing factors. Localization always leads to framing-one results.

Cohomological equivalence

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At classical level, there exists a well defined matrix V and a supercharge Q such that

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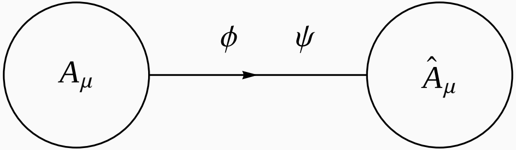
The cohomological equivalence at quantum level leads to

$$\langle W_{\text{bos}} \rangle_1 = \langle W_{1/2} \rangle_1 \tag{12}$$

where the subscript indicates that the identity holds only at framing one.

[HO, Wu, Zhang, 15]

- We consider 3d $\mathcal{N}=2$ Chern-Simons-matter quiver gauge theory with gauge group $U(N)\times U(M)$.
- There are matter fields in bifundamental representations $(N,\bar{M})$.



Fermionic BPS Wilson line along $x^\mu = \tau\delta_0^\mu$

$$W_{\text{fer}} = \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau L_{\text{fer}}(\tau)\right), \quad L_{\text{fer}} = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix},$$

$$\begin{aligned}
&\mathcal{A} = A_\mu \dot{x}^\mu + \sigma + i\alpha\beta\bar{u}u\phi\bar{\phi}, \\
&\hat{\mathcal{A}} = \hat{A}_\mu \dot{x}^\mu + \hat{\sigma} + i\alpha\beta\bar{u}u\bar{\phi}\phi, \\
&\bar{f}_1 = \alpha\bar{u}\psi, \quad f_2 = \beta\bar{\psi}u. \\
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The requirement for W_{fer} to be BPS is [K. Lee, S. Lee, 2010]

$$
\delta L_{\text{fer}} = \partial_\tau \Lambda + \mathrm{i}[L_{\text{fer}}, \Lambda] \tag{14}
$$

for some Grassmann odd matrix

$$
\Lambda = \begin{pmatrix} & \bar{g}_1 \\ g_2 & \end{pmatrix} \tag{15}
$$

Line or circle

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- $W(s,t) = \mathcal{P}\exp\left(-\mathrm{i}\int_s^t \mathrm{d}\tau L_{\mathrm{fer}}(\tau)\right)$, $\delta W(s,t) = -i\Lambda(s)W(s,t) + iW(s,t)\Lambda(t)$

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- Line: $\Lambda(\pm\infty) = 0$, $\delta\left(\mathcal{P}\exp\left(-\mathrm{i}\int_{-\infty}^{\infty} \mathrm{d}\tau L_{\mathrm{fer}}(\tau)\right)\right) = 0$.
  Preserve 2 Poincaré + 2 conformal supercharges, same as the bosonic one.

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  Preserve 2 Poincaré + 2 conformal supercharges, same as the bosonic one.
- Circle: Λ satisfies anti-periodic boundary conditions

$$\Lambda(0) = -\Lambda(2\pi), \tag{16}$$

to construct BPS Wilson loop, we should take the trace

$$\delta\left(\mathrm{Tr}\mathcal{P}\exp\left(-\mathrm{i}\int_0^{2\pi} \mathrm{d}\tau L_{\text{fer}}(\tau)\right)\right) = 0. \tag{17}$$

It is convenient to define a fermionic transformation Q by $\delta = \theta Q$ and $\epsilon = u\theta$. L_{fer} can be compactly written as

$$
\begin{aligned}
&L_{\text{fer}} = L_{\text{bos}} + QG + i\bar{u}uG^2 \\
&L_{\text{bos}} = \begin{pmatrix} \mathcal{A} & 0 \\ 0 & \hat{\mathcal{A}} \end{pmatrix}, \quad G = \begin{pmatrix} 0 & -i\alpha\phi \\ i\beta\phi & 0 \end{pmatrix}
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Cohomological equivalence holds at classical level:

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$$

It was conjectured that cohomological equivalence holds quantum mechanically.

1/6 Fermionic BPS Wilson loops in ABJM theory

$$L_{\text{fer}} = \begin{pmatrix} A_0 + \frac{2\pi}{k} U^I{}_J \phi_I \bar{\phi}^J & \sqrt{\frac{4\pi}{k}} (\bar{\alpha}_I \psi_+^I + \bar{\gamma}_I \psi_-^I) \\ \sqrt{\frac{4\pi}{k}} (\bar{\psi}_{I-} \beta^I - \bar{\psi}_{I+} \delta^I) & B_0 + \frac{2\pi}{k} U^I{}_J \bar{\phi}^J \phi_I \end{pmatrix}$$

$$U^I{}_J = \begin{pmatrix} -1 + 2\beta^2 \bar{\alpha}_2 & -2\beta^1 \bar{\alpha}_2 & & \\ -2\beta^2 \bar{\alpha}_1 & -1 + 2\beta^1 \bar{\alpha}_1 & & \\ & & 1 - 2\delta^4 \bar{\gamma}_4 & 2\delta^3 \bar{\gamma}_4 \\ & & 2\delta^4 \bar{\gamma}_3 & 1 - 2\delta^3 \bar{\gamma}_3 \end{pmatrix}$$

$$\bar{\alpha}_I = (\bar{\alpha}_1, \bar{\alpha}_2, 0, 0), \;\; \beta^I = (\beta^1, \beta^2, 0, 0), \;\; \bar{\gamma}_I = (0, 0, \bar{\gamma}_3, \bar{\gamma}_4), \;\; \delta^I = (0, 0, \delta^3, \delta^4)$$

The Wilson loop is 1/6-BPS, when the parameters satisfy the constraints

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For special parameters in the constructions, the Wilson loops become the 1/2-BPS. For example

$$\bar{\alpha}_2 = \beta^2 = 1, \quad \bar{\alpha}_1 = \beta^1 = \delta^I = \bar{\gamma}_J = 0 \tag{21}$$

Holographic description [Correa, Giraldo-Rivera, Silva, 19]

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- Fermionic $1/2$ BPS Wilson loops $\rightarrow$ Dirichlet boundary condition
- Bosonic $1/6$ BPS Wilson loops $\rightarrow$ Neumann boundary condition

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- Fermionic 1/6 BPS Wilson loops $\rightarrow$ mixed boundary condition

Fermionic BPS Wilson loops in four dimensions

$\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory

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- Let us consider a marginal deformation of the $\mathbb{Z}_2$ orbifold of the $\mathcal{N} = 4$ SYM.
  [Rey, Suyama, 10]

$\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory

- Let us consider a marginal deformation of the $\mathbb{Z}_2$ orbifold of the $\mathcal{N} = 4$ SYM. [Rey, Suyama, 10]
- The fields in the two $\mathcal{N} = 2$ vector multiplets can be arranged into 2×2 block matrices:

$$A_\mu = \begin{pmatrix} A_\mu^{(1)} & 0 \\ 0 & A_\mu^{(2)} \end{pmatrix}, \quad \mu = 0, ..., 5$$
$$\lambda_\alpha = \begin{pmatrix} \lambda_\alpha^{(1)} & 0 \\ 0 & \lambda_\alpha^{(2)} \end{pmatrix}, \quad \alpha = 1, 2, \tag{22}$$

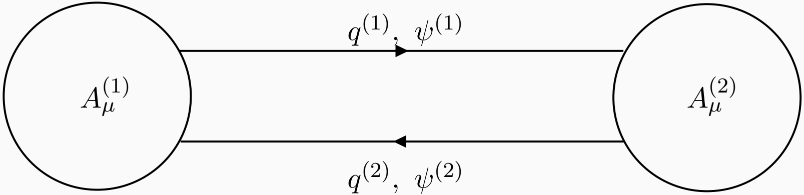
where A_m with $m = 0, ..., 3$ is the gauge field and $A_{4,5}$ are two real scalars. The $SO(1,5)$ Weyl spinors λ_1 and λ_2 have chirality -1 for Γ^{012345} and satisfy the reality condition $\bar{\lambda}^\alpha = -\epsilon^{\alpha\beta}\lambda^c_\beta$.

$\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory

- The matter content consists of two bifundamental hypermultiplets with component fields:

$$q^{\alpha} = \begin{pmatrix} 0 & q^{(1)\alpha} \\ q^{(2)\alpha} & 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} 0 & \psi^{(1)} \\ \psi^{(2)} & 0 \end{pmatrix}, \qquad (23)$$

- Quiver diagram:



$\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory

- The action of the $\mathcal{N}=2$ gauge theory is

$$\begin{aligned} S_{\mathcal{N}=2} \quad = \quad & \int d^4x \left(-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{i}{2}\mathrm{Tr}(\bar{\lambda}^{\alpha}\Gamma^{\mu}D_{\mu}\lambda_{\alpha}) - D_{\mu}q_{\alpha}D^{\mu}q^{\alpha} - i\bar{\psi}\Gamma^{\mu}D_{\mu}\psi \right. \\ & + \sqrt{2}g\bar{\lambda}^{\alpha A}q_{\alpha}T_A\psi - \sqrt{2}g\bar{\psi}T_A q^{\alpha}\lambda_{\alpha}^{A} - g^2(q_{\alpha}T^{A}q^{\beta})(q_{\beta}T_{A}q^{\alpha}) \\ & \left. + \frac{1}{2}g^2(q_{\alpha}T_{A}q^{\alpha})(q_{\beta}T^{A}q^{\beta}) \right) . \end{aligned} \qquad (24)$$

$\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory

- The action of the $\mathcal{N} = 2$ gauge theory is

$$S_{\mathcal{N}=2} = \int d^4x \Big(-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{i}{2}\mathrm{Tr}(\bar{\lambda}^{\alpha}\Gamma^{\mu}D_{\mu}\lambda_{\alpha}) - D_{\mu}q_{\alpha}D^{\mu}q^{\alpha} - i\bar{\psi}\Gamma^{\mu}D_{\mu}\psi + \sqrt{2}g\bar{\lambda}^{\alpha A}q_{\alpha}T_A\psi - \sqrt{2}g\bar{\psi}T_A q^{\alpha}\lambda^A_{\alpha} - g^2(q_{\alpha}T^A q^{\beta})(q_{\beta}T_A q^{\alpha}) + \frac{1}{2}g^2(q_{\alpha}T_A q^{\alpha})(q_{\beta}T^A q^{\beta}) \Big) . \quad (24)$$

- The coupling constants for the two gauge group factors can be varied independently while preserving $\mathcal{N} = 2$ superconformal symmetry. We assemble them into a matrix:

$$g = \begin{pmatrix} g^{(1)} I_N & 0 \\ 0 & g^{(2)} I_N \end{pmatrix} , \quad (25)$$

$\mathcal{N} = 2$ superconformal transformation

The action is invariant under the $\mathcal{N} = 2$ superconformal transformation:

$$\begin{aligned}
\delta A_\mu &= -i\bar{\xi}^\alpha \Gamma_\mu \lambda_\alpha = i\bar{\lambda}^\alpha \Gamma_\mu \xi_\alpha, \\
\delta q^\alpha &= -i\sqrt{2}\bar{\xi}^\alpha \psi, \\
\delta q_\alpha &= -i\sqrt{2}\bar{\psi}\xi_\alpha, \\
\delta \psi &= -\sqrt{2}D_\mu q^\alpha \Gamma^\mu \xi_\alpha - 2\sqrt{2}q^\alpha \vartheta_\alpha, \\
\delta \bar{\psi} &= \sqrt{2}\bar{\xi}^\alpha \Gamma^\mu D_\mu q_\alpha - 2\sqrt{2}\bar{\vartheta}^\alpha q_\alpha, \\
\delta \lambda^A_\alpha &= \frac{1}{2}F^A_{\mu\nu}\Gamma^{\mu\nu}\xi_\alpha + 2igq_\alpha T^A q^\beta \xi_\beta - igq_\beta T^A q^\beta \xi_\alpha - 2A^A_a \Gamma^a \vartheta_\alpha, \\
\delta \bar{\lambda}^{\alpha A} &= -\frac{1}{2}\bar{\xi}^\alpha F^A_{\mu\nu}\Gamma^{\mu\nu} - 2igq_\beta T^A q^\alpha \bar{\xi}^\beta + igq_\beta T^A q^\beta \bar{\xi}^\alpha + 2\bar{\vartheta}^\alpha A^A_a \Gamma^a,
\end{aligned} \tag{26}$$

where $\xi_\alpha = \theta_\alpha + x^m \Gamma_m \vartheta_\alpha$ and the index $a = 4, 5$.

Holographic description

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- dual to type IIB string theory on the $AdS_5 \times (S^5/\mathbb{Z}_2)$ geometry [Kachru, Silverstein, 98]
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- The parameter θ is proportional to the flux of the NSNS B-field through collapsed two-cycle of the orbifold.
  [Lawrence, Nekrasov, Vafa, 98] [Klebanov, Nekrasov, 99]

One can define a 1/2 BPS Wilson line along the timelike infinite straight line $x^m = \delta_0^m \tau$ as

$$W_{\text{bos}} = \mathcal{P} e^{i \int \mathrm{d}\tau L_{1/2}(\tau)}, \quad L_{1/2} = gA_0 - gA_5. \tag{28}$$

The persevered supersymmetries can be parameterized by ξ_α satisfying

$$\Gamma_5 \Gamma_0 \xi_\alpha = \xi_\alpha. \tag{29}$$

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The matrices B and F are defined as

$$B = \begin{pmatrix} B^{(1)} & 0 \\ 0 & B^{(2)} \end{pmatrix}, \tag{32}$$

$$F = \zeta^c \psi + \bar{\psi}\eta, \tag{33}$$

$$\zeta = \begin{pmatrix} \zeta^{(1)} I_N & 0 \\ 0 & \zeta^{(2)} I_N \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^{(2)} I_N & 0 \\ 0 & \eta^{(1)} I_N \end{pmatrix}, \tag{34}$$

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- We demanding L to transform as

$$Q_s L = \mathcal{D}_0 G_s \equiv \partial_0 G_s - i[L_{1/2} + B, G_s] + i\{F, G_s\}. \tag{35}$$

We find

$$L = L_{1/2} + \frac{2i}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} Q_s G_s - \frac{2}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} G_s^2, \tag{36}$$

$$G_s = \zeta^c \Gamma_0 s_\alpha q^\alpha - q_\alpha \bar{s}^\alpha \Gamma_0 \eta, \tag{37}$$

$$\Gamma_5 \Gamma_0 \eta = \eta, \quad \zeta^c \Gamma_5 \Gamma_0 = -\zeta^c \tag{38}$$

Check

$$Q_s \left(\frac{2i}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} Q_s G_s \right) = \partial_0 G_s - i[L_{1/2}, G_s], \tag{39}$$

$$Q_s \left(-\frac{2}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} G_s^2 \right) = i\{ \frac{2i}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} Q_s G_s, G_s \} \tag{40}$$

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- For special parameters in the constructions, it can preserve one more Poincaré supercharge.

Wilson loop preserving two supercharges

We assume the Wilson loop preserves Q_s. When $(\zeta^{(1)}, \eta^{(1)}, \Gamma^{35}\zeta^{(2)*}, \Gamma^{35}\eta^{(2)*})$ are all proportional to each other and s_α can be decomposed as

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and the Wilson loop also preserves Q_u with

$$u_\alpha = \bar{k}\epsilon_{\alpha\beta}\bar{c}^\beta \zeta^{(1)} + k c_\alpha \Gamma^{35}\zeta^{(1)*}, \tag{42}$$

where k is a complex number.

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Fermionic BPS Wilson loops in $\mathcal{N} = 4$ super Yang-Mills theory

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$$L = I_r \otimes L_{1/2} + \frac{r}{s^c \Pi_- \Gamma^5 s} M^S \otimes \mathcal{Q}_s A_S + \frac{ir}{s^c \Pi_- \Gamma^5 s} (M^S \otimes A_S)^2 \tag{45}$$

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Thanks for attentions!