Special flow equation and GKP-Witten relation

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S.Aoki-J.Balog-T.Onogi-SY ArXiv:2204.06855

Holography

Boundary ⟷ **Bulk**

[D.Gabor '47] ['t Hooft '93, Susskind '94]

$$ds^2 \propto \frac{dz^2 + dx^2}{z^2}$$

z

CFT on $R^{1,d-1}$ ⟷ "AdS/CFT" ⟶ Gravity on AdS_{d+1}

[Banks-Douglas-Horowitz-Martinec '98]... [Maldacena '97]

"Weak point"

∃ issue on how to map the scale and the radial coordinate

∃ issue on the map between conformal transformation & isometry

Assume the dual geometry as AdS

0

⟵

- **Wilsonian RG scale** [Heemskerk-Polchinski '10]
- **Relative coordinate in bilocal field** [Das-Jevicki '03]
- **cMERA (entanglement entropy)** [Swingle '09], [Ryu-Takayanagi '06]

⋮

These weak points are resolved by **"Flow equation" approach!?**

Holography

Boundary ⟷ **Bulk**

[D.Gabor '47] ['t Hooft '93, Susskind '94]

$$ds^2 \propto \frac{dz^2 + dx^2}{z^2}$$

Flow equation approach

CFT on $R^{1,d-1}$ **"AdS/CFT"** Gravity on AdS_{d+1}

[Banks-Douglas-Horowitz-Martinec '98]... [Maldacena '97]

Vacuum $|0\rangle$ → AdS geometry $g_{MN} = \frac{\eta_{MN}}{z^2}$

[Aoki-SY 2017]

Excited state $|\mathcal{O}\rangle$ → ???

[Aoki-Balog-Onogi-SY 2022]

Today!

z

0

Plan

✓ 1. Introduction

2. Flow equation & dual geometry

3. Special flow equation

4. Correlation function & bulk reconstruction

5. Summary

Flow equation

Flow equation

1. was introduced to help numerics of lattice QCD.

cf. def of stress energy tensor

[Albanese et al. (APE) '87] [Narayanan-Neuberger '06] [Luscher '10,'13]

2. describes a **non-local course-graining of an operator**.

Consider a CFT_d which contains a primary scalar ϕ

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{1}{x_{12}^{2\Delta}}$$

$$x_{12} := x_1 - x_2$$

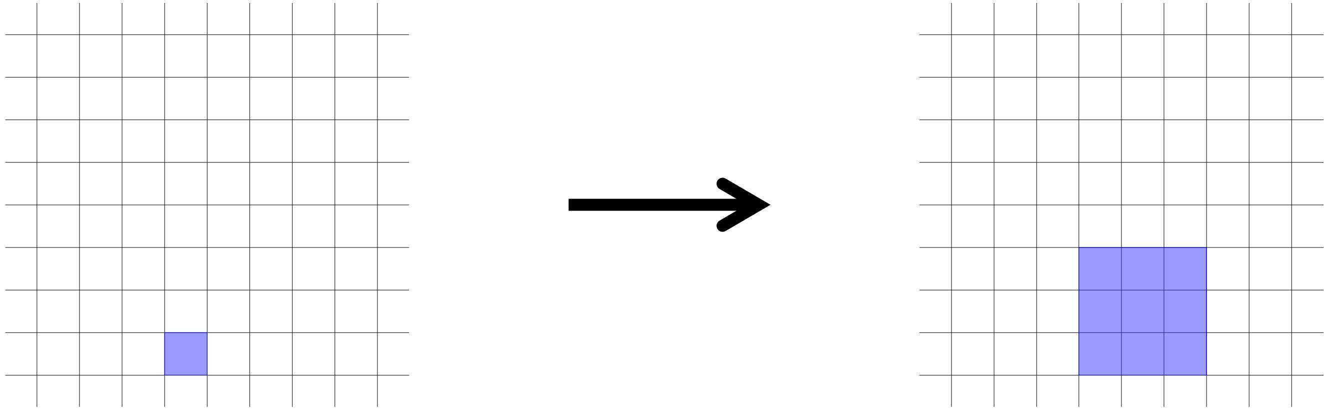
Free flow equation

$$\frac{\partial \phi(x;\eta)}{\partial \eta} = \partial^2 \phi(x;\eta). \quad \phi(x;0) = \phi(x)$$

The solution: $$\phi(x;\eta) = \int d^d y\, K(x-y;\eta)\phi(y). \qquad K(x-y;\eta) = \frac{e^{-(x-y)^2/4\eta}}{(4\pi\eta)^{d/2}}$$

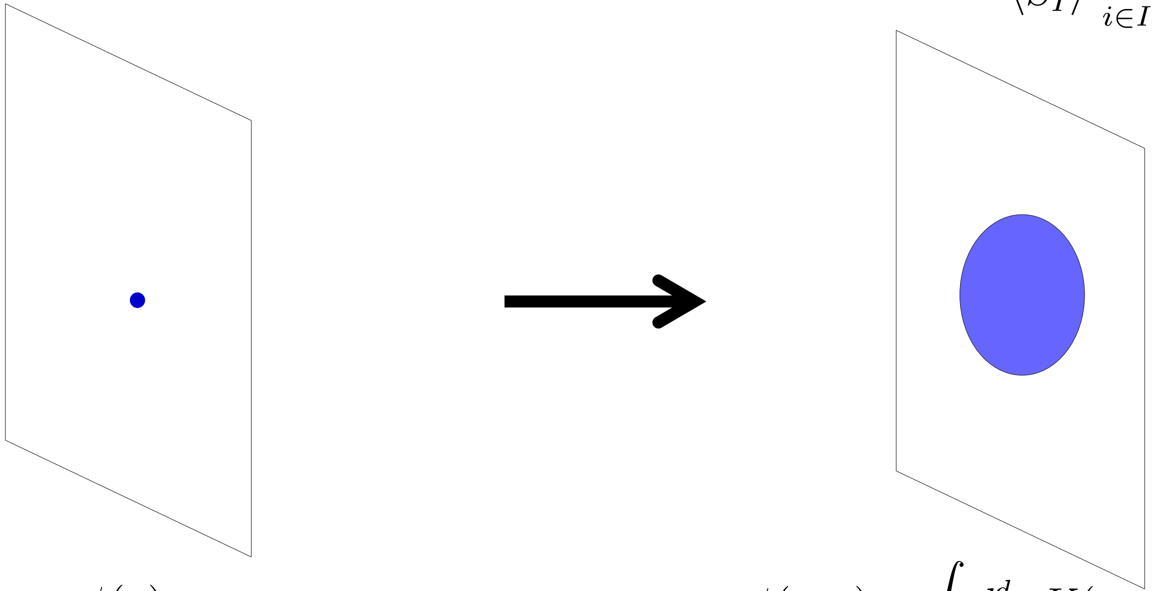
➡ Reminiscent of the block spin transformation!?

Block spin v.s. Flowed operator



s_i

$$S_I = \frac{1}{\langle S_I \rangle} \sum_{i \in I} s_i$$



$\phi(x)$

"Point particle"

$$\phi(x;\eta) = \int d^d y \, K(x-y;\eta)\phi(y).$$

"Gaussian wave packet"

Flow equation

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Claim: **Contact singularity in 2pt function is resolved**.

[Albanese et al. (APE) '87] [Narayanan-Neuberger '06]

$$\langle \phi(x_1;\eta_1)\phi(x_2;\eta_2)\rangle = \frac{1}{\eta_+^{\Delta}} F\Big(\frac{x_{12}^2}{\eta_+};1\Big) \qquad \eta_+ := \eta_1 + \eta_2$$

$$F(v;1) = \frac{1}{(4\pi)^{\frac{d}{2}}} \int_0^1 du (1-u)^{d/2-\Delta-1} e^{-vu/4} u^{\Delta-1} \qquad \frac{d-2}{2} \leq \Delta < \frac{d-1}{2}$$

Construction of holographic space

[Aoki-SY ’17][Aoki-Balog-SY ’18]

Metric operator and holographic metric

[Aoki-Kikuchi-Onogi '15] [Aoki-Balog-Onogi-Weisz '16,'17]

Def. (Normalized operator)

$$\sigma^a(x;\eta) = \frac{\phi^a(x;\eta)}{\sqrt{\sum_{b=1}^N \phi^b(x;\eta)^2}}$$

"Operator renormalization"

$$\Rightarrow \langle \sigma^a(x;\eta)\, \sigma^b(x;\eta) \rangle = \frac{1}{N}\delta^{ab}$$

$$\Rightarrow \langle \sigma^a(x;\eta)\, \sigma^a(x;\eta) \rangle = 1$$

NOTE This is well-defined due to the absence of the contact singularity.

Def. (Metric operator)

$$\hat{g}_{MN}(x;\eta) = \Sigma_{a=1}^N \frac{\partial}{\partial X^M}\sigma^a(x;\eta) \frac{\partial}{\partial X^N}\sigma^a(x;\eta)$$

$$(X^M) = (x^\mu, z) \; with \; z := \sqrt{\eta/\gamma}$$

↓

Def. (Metric of the "holographic space")

$$g_{MN}(X) = \langle \hat{g}_{MN}(x;\eta) \rangle$$

"Why do you construct like this?"

[Aoki-SY '17]

① The induced metric can be interpreted as the Bure **information metric**.

The Bure **information metric** measures the distance of 2 quantum states:

Def. (Bure information metric)

$$D(\rho,\rho+d\rho)^2=\frac{1}{2}tr(d\rho H) \qquad \rho H+H\rho=d\rho$$

ρ: density matrix ($\mathrm{tr}(\rho)=1$) $\qquad H$: Hermitian 1 − form operator

$$\rho_X=\Sigma_{a=1}^N|\sigma^a(x;\eta)\rangle\langle\sigma^a(x;\eta)|$$

where we define the bra and ket states by

$$|\sigma^a(x;\eta)\rangle:=\sigma^a(x;\eta)|0\rangle, \quad \langle\sigma^a(x;\eta)|:=\langle 0|\sigma^a(x;\eta)$$

NOTE: This is the usual Hermitian conjugate.

$$H_X=Nd\rho_X$$

($\because$ ρ_X is a pure state up to a scale factor: $\rho_X^2=\frac{1}{N}\rho_X$.)

$$D(\rho_X,\rho_X+d\rho_X)^2=\frac{N}{2}tr(d\rho_X d\rho_X)=\frac{N}{2}tr(\partial_M\rho\partial_N\rho)dX^MdX^N$$

$$=\frac{N}{2}\Sigma_{a,b=1}^N tr\big(\partial_M(|\sigma^a(x;\eta)\rangle\langle\sigma^a(x;\eta)|)\partial_N(|\sigma^b(x;\eta)\rangle\langle\sigma^b(x;\eta)|)\big)dX^MdX^N$$

$$=\Sigma_{a=1}^N(\langle\partial_N\sigma^a(x;\eta)|\partial_M\sigma^a(x;\eta)\rangle)dX^MdX^N=g_{MN}dX^MdX^N$$

"Why do you construct like this?"

[Aoki-Balog-SY '18]

② Classical geometry appears in the leading term in the 1/N expansion.
Bulk quantum correction is encoded into the subleading terms in the 1/N expansion.

Def. (pre-geometric operator)

$$\mathcal{O}[g] \to \mathcal{O}[\hat{g}] =: \hat{\mathcal{O}}$$

$\mathcal{O}$: geometric object in differential geometry

ex.

$$\hat{\Gamma}^M_{LN}(x;\eta) = \frac{1}{2}\hat{g}^{MP}(x;\eta)(\hat{g}_{P\{N,L\}}(x;\eta) - \hat{g}_{NL,P}(x;\eta))$$

$$\hat{R}_{LP}{}^{M}{}_{N}(x;\eta) = \partial_{[L}\hat{\Gamma}^M_{P]N}(x;\eta) + \hat{\Gamma}^M_{[LQ}(x;\eta)\hat{\Gamma}^Q_{P]N}(x;\eta)$$

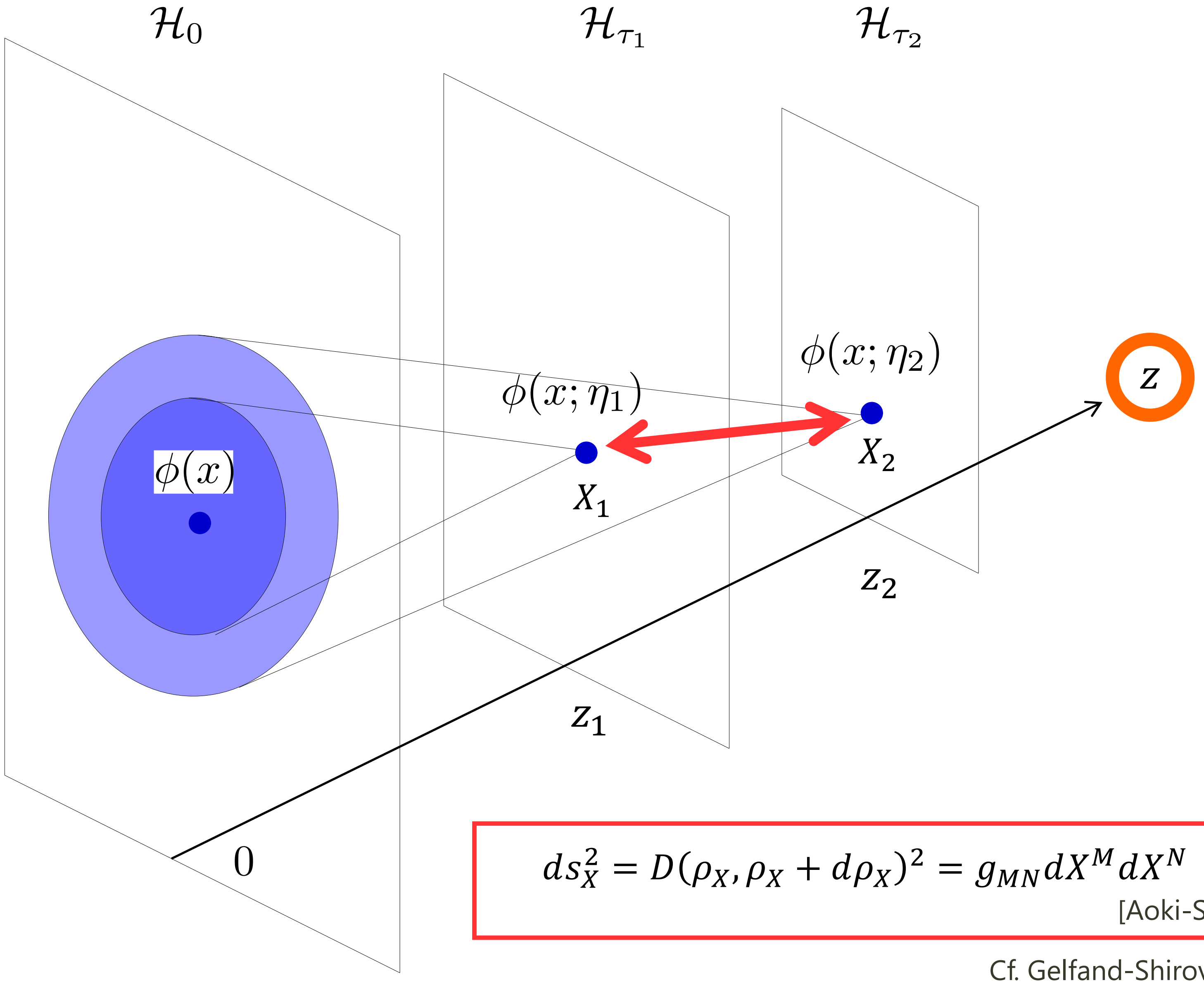
⋮

$$\Rightarrow \quad \langle\mathcal{O}[\hat{g}]\rangle = \mathcal{O}[\langle\hat{g}\rangle] + \langle\mathcal{O}[\hat{g}]\rangle_c$$

Leading Order (LO) ↕ Classical geometry

Next to Leading Order (NLO) ↕ Quantum correction

Smearing = Extra direction



$$ds_X^2 = D(\rho_X, \rho_X + d\rho_X)^2 = g_{MN}dX^M dX^N$$

[Aoki-SY '17]

Cf. Gelfand-Shirov thm

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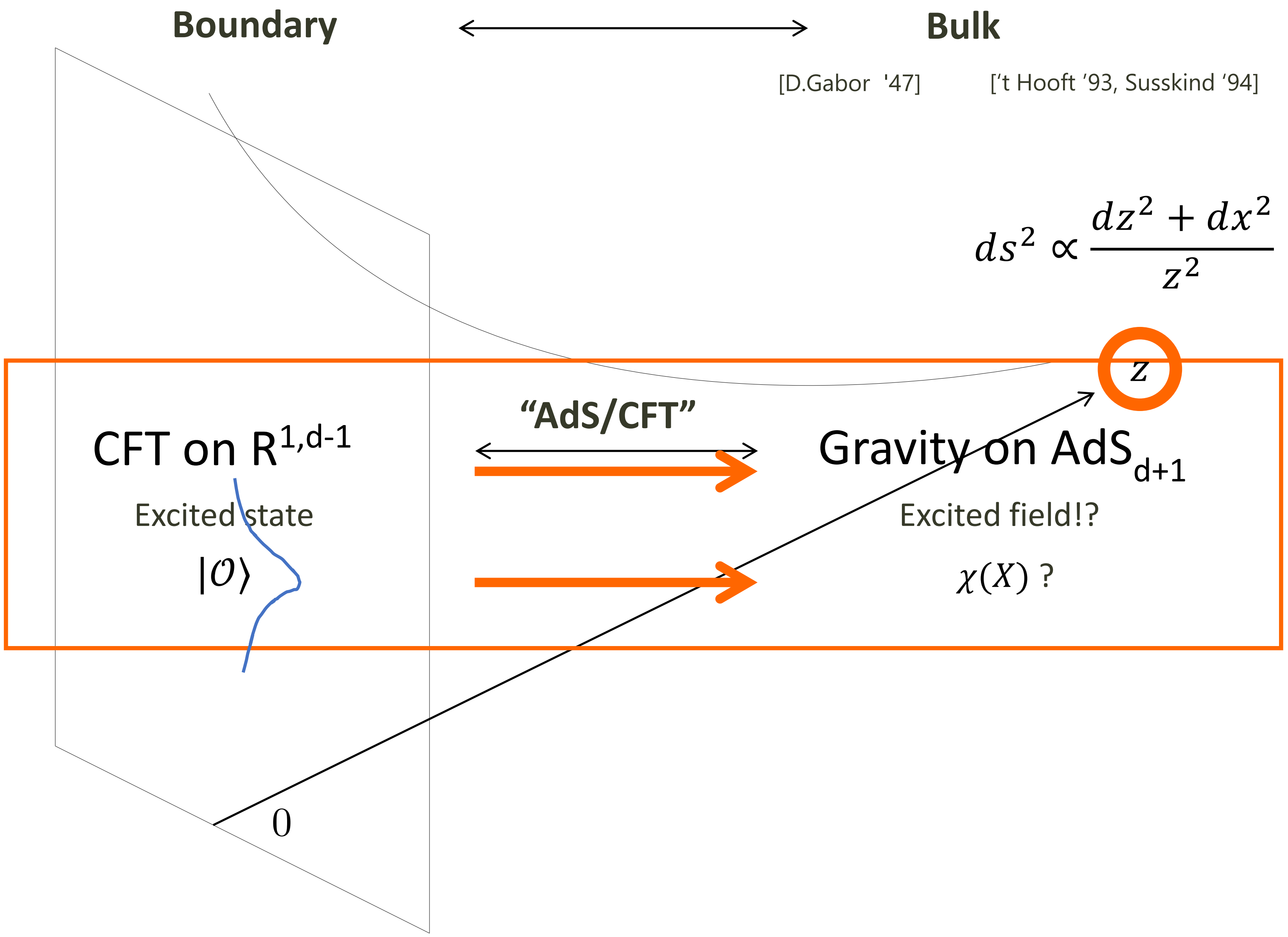
Special flow equation

[Aoki-Balog-Onogi-SY ’22]

Holography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"AdS/CFT"
CFT on $R^{1,d-1}$
Gravity on AdS_{d+1}
Excited state
$|\mathcal{O}\rangle$
Excited field!?
$\chi(X)$?
0

Bulk construction for excited states?

① Model CFT

→ Free O(N) vector model

$$\phi^a(x) \qquad \langle \phi^a(x_1)\phi^b(x_2)\rangle = \frac{C\delta^{ab}}{x_{12}^{2\Delta}}$$

② CFT Excited state

→ O(N) singlet state

$$|O(y)\rangle = O(y)|0\rangle \qquad O(y) = \frac{1}{N}:(\phi^a)^2:(\mathrm{y})$$

③ Expected excitation of a scalar field χ in the bulk!?

$$\chi(X) = \langle 0| \sum_{a=1}^{N} \sigma^a(x;\eta)^2 \, |O(y)\rangle$$

→ Needs to satisfy the (constructed) bulk equation of motion.

$$(\nabla^2_{AdS} - m^2)\chi(X) = 0?$$

$$\nabla^2_{AdS} = z^2(\partial_z^2 + \partial^2) - (d-1)z\partial_z, \qquad m^2 = \Delta_O(\Delta_O - d)$$

↓

But it does not satisfy… $(\nabla^2_{AdS} - m^2)\chi(X) \neq 0$ → Something should be modified…

Interpolating flow equation

→ Consider a minimal extension of the free flow equation.

$$\frac{\partial \phi^a(x;\eta)}{\partial \eta} = \partial^2 \phi(x;\eta) \rightarrow (-\alpha \frac{\partial^2}{\partial \eta^2} + \beta \frac{\partial}{\partial \eta})\phi^a(x;\eta) = \partial^2 \phi(x;\eta)$$

Solution

$$\rightarrow \left(-\alpha \frac{\partial^2}{\partial \eta^2} + \beta \frac{\partial}{\partial \eta}\right)\rho_\eta(p) = -p^2 \rho_\eta(p)$$

$$\phi^a(x;\eta) = \int d^d y\, \rho_\eta(x-y)\varphi^a(y)$$

$$\rho_0(p) = 1$$

$$\rightarrow \rho_\eta(p) = \frac{2}{\Gamma(\nu)} \tilde{p}^\nu K_\nu(2\tilde{p}) \qquad \nu = (\alpha+\beta)/\alpha \qquad \tilde{p} = \sqrt{\frac{\eta}{\alpha}}p, \qquad p = \sqrt{p^2}$$

Modified Bessel function

$$\rho_\eta(x) = \frac{2}{\Gamma(\nu)} 2\pi^{\frac{d}{2}} \left(\frac{x}{2}\right)^{1-\frac{d}{2}} \int_0^\infty \frac{dp p^{\frac{d}{2}}}{(2\pi)^d} J_{\frac{d}{2}-1}(px)\tilde{p}^\nu K_\nu(2\tilde{p})$$

$$\overset{(\nu < \frac{d}{2})}{=} \frac{\Gamma\left(\nu + \frac{d}{2}\right)}{\Gamma(\nu)} \left(\frac{\alpha}{4\pi\eta}\right)^{\frac{d}{2}} \left(1 + \frac{\alpha x^2}{4\eta}\right)^{-\nu-\frac{d}{2}}$$

$$\int_0^\infty dk k^{\mu+\nu+1} J_\mu(xk) K_\nu(bk) = \frac{(2x)^\mu (2b)^\nu}{(x^2+b^2)^{\mu+\nu+1}} \Gamma(\mu+\nu+1)$$

with $\mathrm{Re}[b] > |\mathrm{Im}[a]|, \mathrm{Re}[\mu] > |\mathrm{Re}[\nu]| - 1.$

NOTE $\rho_\eta(x) \rightarrow \delta^d(x)$ *as long as* $0 < \nu$.

In the $\alpha \rightarrow 0$ limit, $\rho_\eta(x)\Big|_{\beta=1} \rightarrow \left(\frac{1}{4\pi\eta}\right)^{\frac{d}{2}} e^{-\frac{x^2}{4\eta}}$

Smearing function of the free flow

Conformal transformation & AdS isometry

→ Consider an infinitesimal transformation

$$\delta^{conf}\varphi^a(x) = -\delta x^\mu \partial_\mu \varphi(x) - \frac{\Delta}{d}(\partial_\nu \delta x^\nu)\varphi^a(x)$$

$$\delta x^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu (b\cdot x)$$

$$\rightarrow \delta^{conf}\sigma^a(x;\eta) = \frac{1}{\sqrt{\langle \phi^2(x;\eta)\rangle}} \int d^d y\, \rho_\eta(x-y)\delta^{conf}\varphi^a(y)$$

$$= \delta^{diff}\sigma^a(x;\eta) + \delta^{res}\sigma^a(x;\eta)$$

$$\left\{ \begin{aligned} &\delta^{diff}\sigma^a(x;\eta) := -\bar{\delta} x^\mu \partial_\mu \sigma^a(x;\eta) - \bar{\delta} z \partial_z \sigma^a(x;\eta) \\ &\qquad \bar{\delta} x^\mu = \delta x^\mu + \frac{4\gamma}{\alpha} z^2 b^\mu \quad \bar{\delta} z = (\lambda - 2b\cdot x)z \\ &\delta^{res}\sigma^a(x;\eta) := \frac{2\left(\nu - \frac{d}{2} + \Delta\right)}{\sqrt{\langle \phi^2(x;\eta)\rangle}} \int d^d y\, \rho_\eta(y)(b\cdot y)\varphi^a(x-y) \end{aligned} \right.$$

$$z = \sqrt{\eta/\gamma}$$

$$\delta^{res}\sigma^a(x;\eta) = 0 \Leftrightarrow \nu = \frac{d}{2} - \Delta$$

With this parametrization and $\gamma = \alpha/4$, the conformal transformation matches the AdS isometry!

$$\rightarrow \delta^{conf}\sigma^a(x;\eta) = \delta^{diff}\sigma^a(x;\eta) = -\bar{\delta} x^\mu \partial_\mu \sigma^a(x;\eta) - \bar{\delta} z \partial_z \sigma^a(x;\eta)$$

$$\bar{\delta} x^\mu = \delta x^\mu + z^2 b^\mu, \quad \bar{\delta} z = (\lambda - 2b\cdot x)z$$

Conformal transformation & AdS isometry

Claim The induced metric becomes the AdS one.

$\because$ Induced metric is invariant under any conformal transformation

$$\delta^{conf} g_{MN}(X) = \langle \delta^{conf} \hat{g}_{MN}(x;\eta) \rangle = 0$$

On the other hand, the conformal transformation is the same as the AdS isometric one:

$$\delta^{conf} \hat{g}_{MN}(x;\eta) = \delta^{diff} \hat{g}_{MN}(x;\eta)$$

Since the AdS space is a maximally symmetric one, $g_{MN}(X)$ has to be the AdS metric. **Q.E.D.**

The normalized smeared operator is written as

$$\sigma^a(x;\eta) = \int d^d y\, K(X,y)\, \varphi^a(y) \qquad K(X,y) = \frac{\rho_\eta(x-y)}{\sqrt{\langle \phi^2(x;\eta) \rangle}} \propto \left(\frac{z}{z^2 + (x-y)^2} \right)^{d-\Delta}$$

Comment

This kernel is formally the same as the boundary-to-bulk propagator. [Witten '98]

However, in the flow equation approach, the smeared objects are elementary fields, while in other standard ones smeared objects are singlet or gauge-invariant.

$$\Delta < \frac{d}{2} : \text{flow equation approach} \qquad d-1 \leq \Delta : \text{standard approach}$$

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Correlation functions

[Aoki-Balog-Onogi-SY '22]

Constraint for correlation functions

The specially flowed operator $\sigma^a(x;\eta)$ enables us to construct composite operators with bulk indices such that it behaves as a (n-th) tensor in holographic space.

$$G_{M_1 M_2 \cdots M_n}[\sigma](X)$$

$$G_{M_1' M_2' \cdots M_n'}[\sigma](X') = \frac{\partial X^{M_1}}{\partial X'^{M_1'}} \frac{\partial X^{M_2}}{\partial X'^{M_2'}} \cdots \frac{\partial X^{M_n}}{\partial X'^{M_n'}} G_{M_1 M_2 \cdots M_n}[\sigma](X)$$

X': (finite) isometric transformation of X

ex Metric operator $\hat{g}_{MN}(X)$

Claim Correlation functions of operators with bulk indices and primary operators in CFT is severely constrained by conformal symmetry.

$$\begin{aligned} &\langle G_{M_1' M_2' \cdots M_n'}[\sigma](X') T_{\mu_1 \mu_2 \cdots \mu_m}(y') \rangle \\ &= \frac{\partial X^{M_1}}{\partial X'^{M_1'}} \frac{\partial X^{M_2}}{\partial X'^{M_2'}} \cdots \frac{\partial X^{M_n}}{\partial X'^{M'_n}} \times J(y)^{-\Delta_T} \frac{\partial y^{\nu_1}}{\partial y'^{\mu_1}} \frac{\partial y^{\nu_2}}{\partial y'^{\mu_2}} \cdots \frac{\partial y^{\nu_m}}{\partial y'^{\mu_m}} \\ &\times \langle G_{M_1 M_2 \cdots M_n}[\sigma](X) T_{\nu_1 \nu_2 \cdots \nu_m}(y) \rangle \end{aligned}$$

where $J(y) = |\det(\partial_\nu y'^\mu)|^{\frac{1}{d}}$ y': (finite) conformal transformation of y Δ_T: dimension of T

Correlation functions of smeared operators

A correlation function of a smeared operator and an unsmeared one has fixed position dependence:

$$\langle \sigma^a(x;\eta)\varphi^b(y)\rangle \propto \delta^{ab}\left(\frac{z}{(x-y)^2+z^2}\right)^{\Delta}$$

(**Cf.** In the embedding formalism, $\langle \sigma^a(x;\eta)\varphi^b(y)\rangle \propto \dfrac{\delta^{ab}}{\left(-2\hat{X}(X)\cdot\hat{X}(y)\right)^{\Delta}}$

$$\hat{X}^A(X) = \left(\frac{1}{z},\frac{z^2+x^2}{z},\frac{x^\mu}{z}\right) = \frac{1}{z}\hat{X}^A(x) + (0,z,0)$$

'Poincare section')

$$\langle \sigma^a(x_1;\eta_1)\sigma^b(x_2;\eta_2)\rangle = \int d^d y\, K(X_2,y)\,\langle \sigma^a(x_1;\eta_1)\varphi^b(y)\rangle$$

$$\propto \delta^{ab}\int d^d y \left(\frac{z_2}{(x_2-y)^2+z_2^2}\right)^{\mathrm{d}-\Delta}\left(\frac{z_1}{(x_1-y)^2+z_1^2}\right)^{\Delta}$$

(**Cf.** $\displaystyle\propto \sum_{c=1}^{N}\int d^d y\, \langle \varphi^a(x_1;\eta_1)|\varphi^c(y)\rangle\langle(\varphi^c)^*(y)|\varphi^b(x_2;\eta_2)\rangle$

'shadow operator')

$$= \frac{\delta^{ab}}{N}\,{}_2F_1\!\left(\frac{\Delta}{2},\frac{d-\Delta}{2};\frac{d+1}{2};1-\frac{\mathcal{R}^2}{4}\right)$$

$$\mathcal{R} = \frac{(x_1-x_2)^2+z_1^2+z_2^2}{z_1 z_2}\ \left(=-2\hat{X}(X_1)\cdot\hat{X}(X_2)\right):\mathrm{SO}(1,\mathrm{d}+1)\text{ invariant ratio}$$

Note This shows the absence of the contact divergence of 2pt function of smeared operators.

Bulk reconstruction for excited states

[Aoki-Balog-Onogi-SY '22]

Bulk construction for excited states?

① Model CFT

→ Free O(N) vector model

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② CFT Excited state

→ O(N) singlet state

$$|O(y)\rangle = O(y)|0\rangle \qquad O(y) = \frac{1}{N}:(\phi^a)^2:(y)$$

③ Expected excitation of a scalar field χ in the bulk!?

$$\chi(X) = \langle 0| \sum_{a=1}^{N} \sigma^a(x;\eta)^2 \,|O(y)\rangle$$

➡ Needs to satisfy the (constructed) bulk equation of motion.

$$\left(\nabla_{AdS}^2 - m^2\right)\chi(X) = 0?$$

$$\nabla_{AdS}^2 = z^2(\partial_z^2 + \partial^2) - (d-1)z\partial_z, \qquad m^2 = \Delta_O(\Delta_O - d)$$

Correlation functions of smeared operators

A correlation function of a smeared operator and the singlet has fixed position dependence:

$$\chi(X) = \left\langle \sum_{a=1}^{N} \sigma^a(x;\eta)^2\, O(y) \right\rangle \propto \left(\frac{z}{(x-y)^2 + z^2} \right)^{\Delta_O} \propto K(X,y)\Big|_{\Delta \to d - \Delta_O}$$

$\Delta_O = 2\Delta$: conformal dimension of O

This satisfies the (constructed) bulk equation of motion!

$$(\nabla^2_{AdS} - m^2)\chi(X) = 0$$

$$\nabla^2_{AdS} = z^2(\partial_z^2 + \partial^2) - (d-1)z\partial_z, \qquad m^2 = \Delta_O(\Delta_O - d)$$

To reproduce the GKP-Witten relation, couple to a sufficiently small source field J(y)

Def. $$\chi_J(X) := \langle 0| \sum_{a=1}^{N} \sigma^a(x;\eta)^2 \int d^d y\, J(y)\, O(y) |0\rangle$$

Claim. $$(\nabla^2_{AdS} - m^2)\chi(X) = 0 \quad \text{with} \quad \chi_J(X) \underset{z \sim 0}{\sim} z^{\Delta_O} \langle O(y) \rangle_J$$

Proof. $$\chi_J(X) \sim \int d^d y\, J(y) \left(\frac{z}{(x-y)^2} \right)^{\Delta_O} = z^{\Delta_O} \int d^d y\, J(y) \langle O(x) O(y) \rangle$$

$$= z^{\Delta_O} \left\langle O(x) e^{\int d^d y\, J(y) O(y)} \right\rangle = z^{\Delta_O} \langle O(y) \rangle_J$$

Q.E.D.

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CFT on $R^{1,d-1}$

Excited state

$|\mathcal{O}\rangle$

0

"AdS/CFT"

Pre-geometric operator

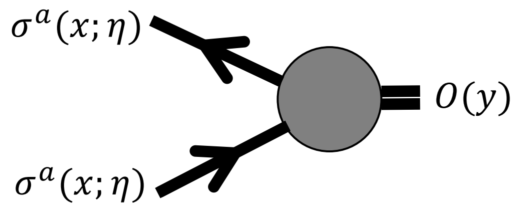
$\sigma^a(x;\eta)^2$

Gravity on AdS_{d+1}

Excited field

$$\chi(X) = \langle 0| \sum_{a=1}^{N} \sigma^a(x;\eta)^2 |O\rangle$$

Nambu-Bethe-Salpter (NBS) wave function



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Summary

- We developed a framework to construct the bulk theory holographically dual to a CFT incorporating **a special flow equation** including excited states.
- A bulk field is realized as **NBS wave function** of a pre-geometric operator and a CFT excited state.
- We succeeded in constructing the dual AdS geometry and a bulk scalar field satisfying the GKP-Witten relation **concurrently**.

Future directions

- For interacting theory? Double-trace deformation? N=4 SYM?
- Minkowski space? Locality in the bulk? Bulk causality? cf. [Hamilton-Kabat-Lifshitz-Lowe '06]
- 1-loop calculation of dual gravity (higher-spin)? cf. [Giombi-Klebanov '02]...
- Finite temperature? BH?

⋮

Thank you!