Love Numbers From Amplitudes

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Based on 2208.08459, 2209.14324, ++ to appear

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HEP Talk@Joint HEP-TH Seminar

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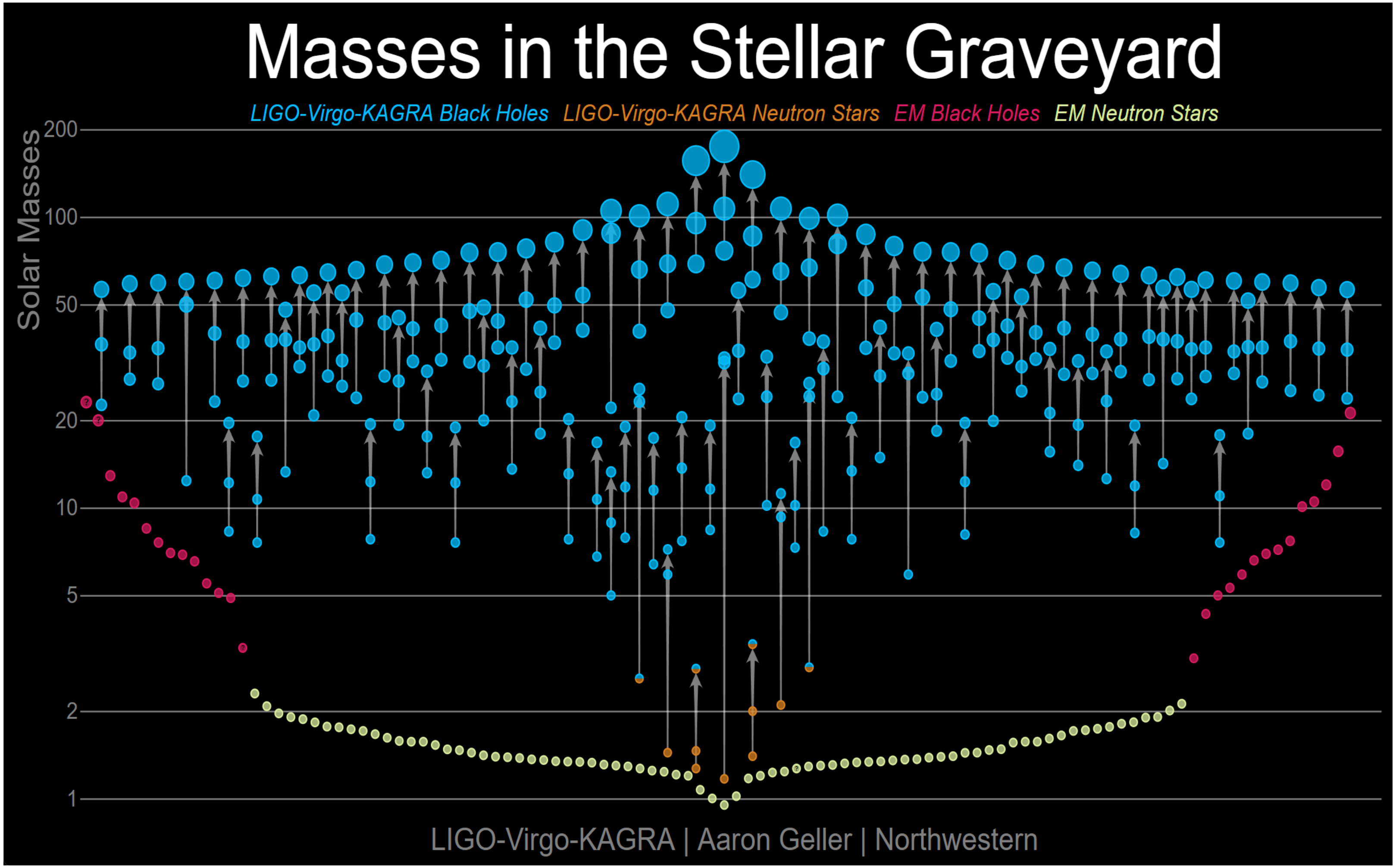
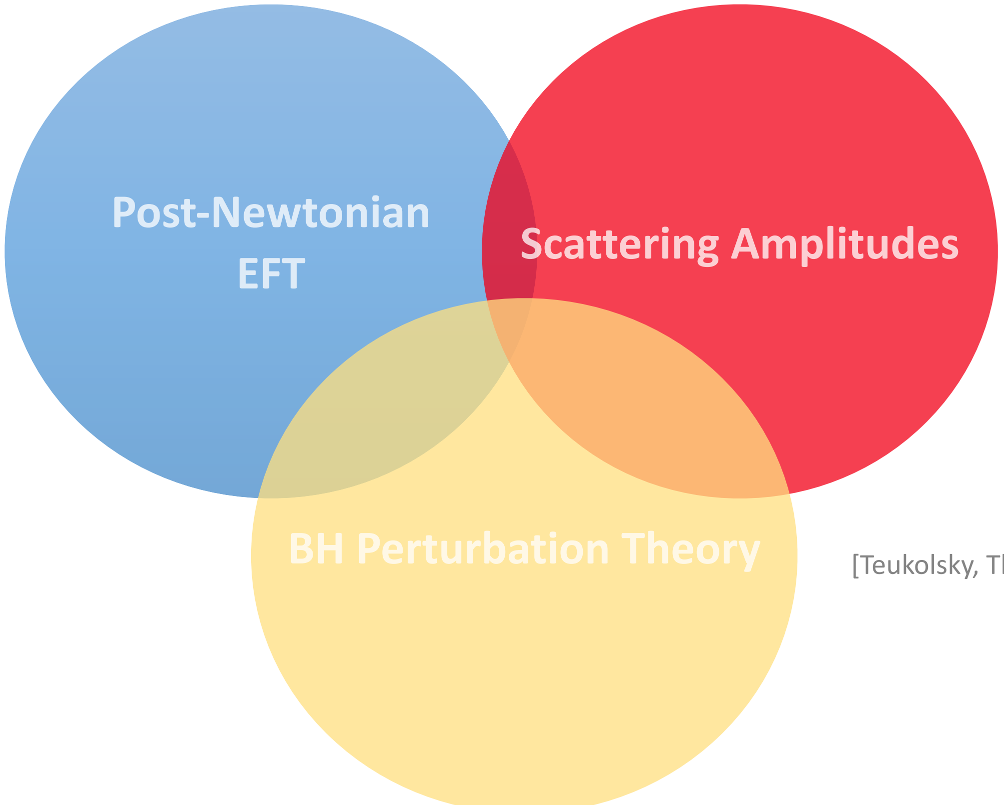
Golden Time For Black Holes Physics



LIGO Virgo Collaboration

Theoretical Study of BH Dynamics

[Goldberger, Rothstein, Porto, Kol, Smolkin, …]



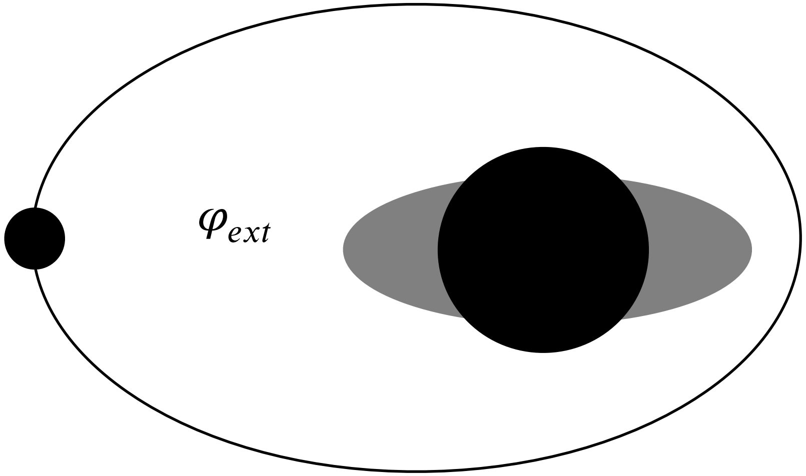
[Bern, Cheung, Roiban, Shen, Solon, Parra-Martinez, …]

[Teukolsky, Thorne, Chen, Pretorius, …]

- Unified in the framework of EFT
- Different structures are manifest in different regions
- Compute the same quantity (e.g. Love number) for cross-check

Tidal Effects

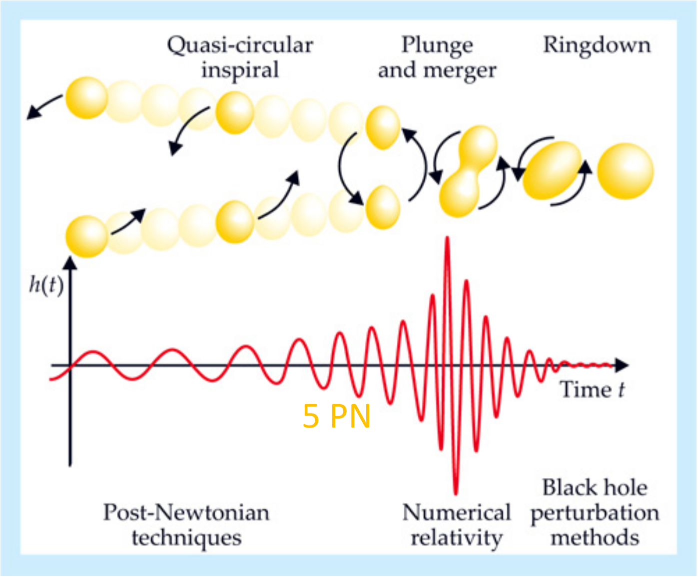
Tidal Deformation:



Possible Observation:

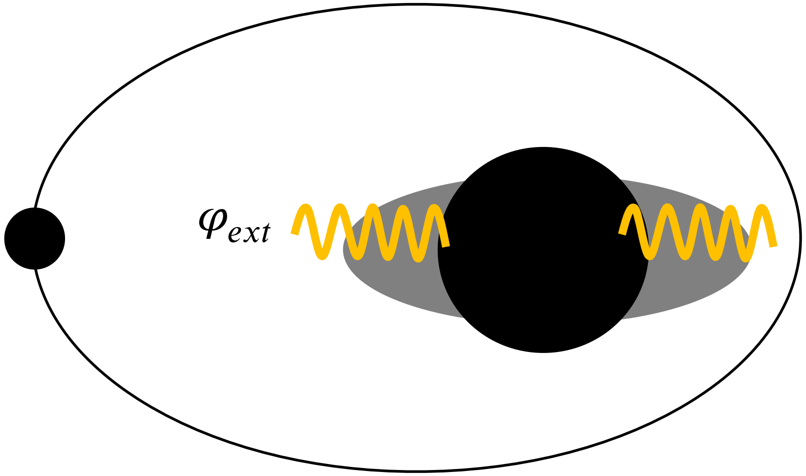
5 PN effects in the phase of GWs

[Flanagan, Hinderer (2007)]



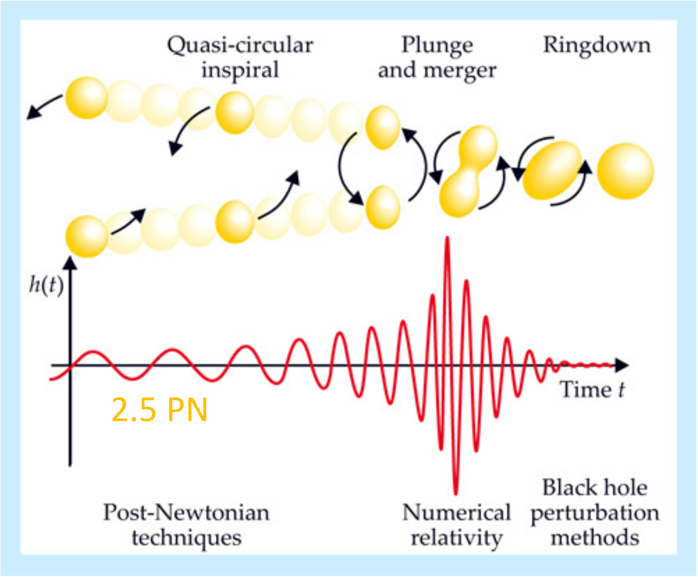
Tidal Effects

Tidal Dissipation:



Possible Observation:

4 PN effects in the phase of
GWs for Schwarzschild BH;
2.5 PN effects for Kerr



[Poisson, Sasaki (1994)]
[Tagoshi, Mano, Takasugi (1997)]

Plan of the Talk

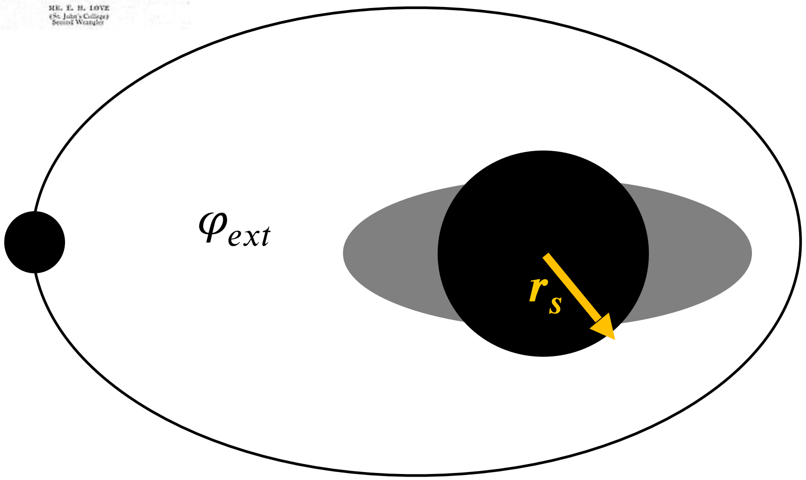
- Tidal deformation and dissipation in GR

- EFT description of Love and dissipation numbers

- BH near zone scattering and vanishing Love

- “Hidden” Love symmetry

Love Numbers in Newtonian Gravity



Quadrupole source and response

$$\varphi(\boldsymbol{x}) = \mathscr{E}_{ij}x^i x^j + \frac{Q_{ij}x^i x^j}{r^5}$$

tilde source   induced quadrupole moments

[Poisson, Will Textbook]

Higher multipole source and response

$$\varphi(\boldsymbol{x}) = \mathscr{E}_{i_1 \cdots i_\ell}x^{i_1}\cdots x^{i_\ell} + Q_{i_1 \cdots i_\ell}\frac{x^{i_1}\cdots x^{i_\ell}}{r^{2\ell+1}}$$

tilde source   induced multiple moments

Linear response theory:

$$Q_\ell = \lambda_\ell r_s^{2\ell+1}\mathscr{E}_\ell - \nu_\ell r_s^{2\ell+2}\frac{d}{dt}\mathscr{E}_\ell + \cdots$$

Love number   Dissipation number

Love Numbers in GR

$+$

[Fang, Lovelace (2005)]
[Binnington, Poisson (2009)]
[Damour, Nagar (2009)]

$$\varphi(\boldsymbol{x}) = \mathcal{E}_{ij}x^i x^j\left(1 + c_1\frac{r_s}{r} + \cdots\right) + \frac{Q_{ij}x^i x^j}{r^5}(1 + \cdots)$$

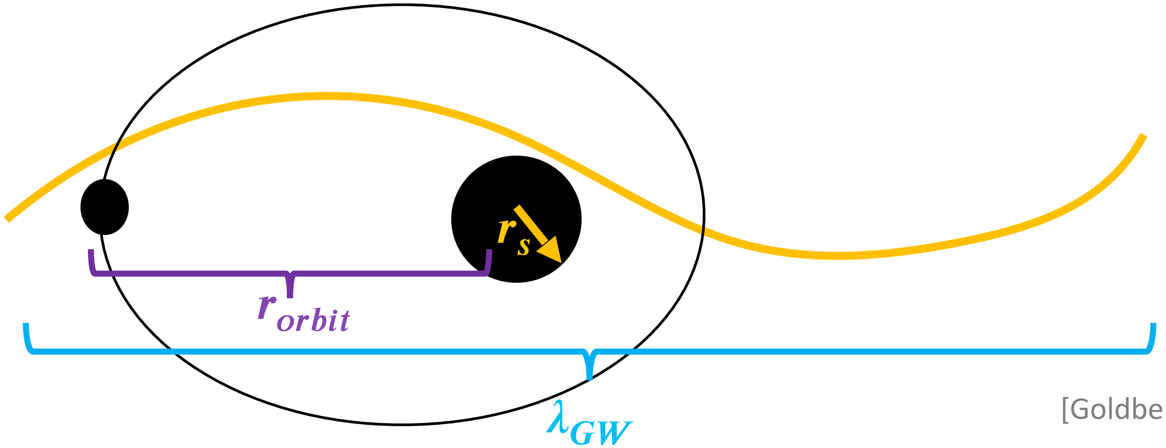
Linear response theory: $Q_\ell = \lambda_\ell r_s^{2\ell+1}\mathcal{E}_\ell - \nu_\ell r_s^{2\ell+2}\frac{d}{dt}\mathcal{E}_\ell + \cdots$

conservative — dissipative

Ambiguity:

- Nonlinear GR: source/response mixing?
- Gauge invariance of Love numbers?

Worldline EFT



[Goldberger, Rothstein (2004)]

Method of regions:

Finite Size Effects	Potential Region	Radiation Region
r_s	r_{orbit}	λ_{GW}
$m,\ P_i,\ Q_{ij}\ldots$	$\partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu},\ \partial_0 H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}$	$\partial_\alpha h_{\mu\nu} \sim \frac{v}{r} h_{\mu\nu}$

Finite Size Effective Action

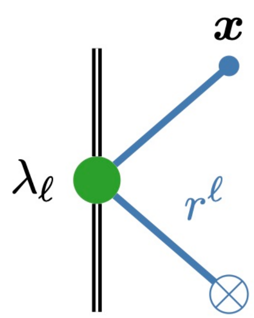
Conservative Dynamics

Let's consider a toy mode of spin-0 fields:

Schwarzschild: $$S = -m\int d\tau + \lambda_1 r_s^3 \int d\tau (\partial_i \phi)^2 + \lambda_2 r_s^5 \int d\tau (\partial_{\langle i}\partial_{j\rangle}\phi)^2 + \cdots$$

[Kol, Smolkin (2020)]
[Hui, Joyce et al (2020)]
[Ivanov, **Zhou** (2022)]

$$\phi = \mathcal{E}_\ell r^\ell + \phi_{\text{resp}}$$



$$\sim \mathcal{E}_\ell r^\ell \times \lambda_\ell \left(\frac{r_s}{r}\right)^{2\ell+1}$$

Kerr: $$S = -m\int d\tau + \frac{1}{2}\int d\tau S^{\mu\nu}\Omega_{\mu\nu} + \cdots + r_s^3 \int d\tau \lambda^{ij}(\partial_i\phi\partial_j\phi) + \cdots$$

[Le Tiec et al (2020)]
[Charalambous, Dubovsky, Ivanov (2021)]

Love numbers to Love tensors

* Love numbers = Wilson coefficients! (gauge-invariant definition)

Finite Size Effective Action

Dissipative Dynamics

$$S \supset \int d\tau Q^i(X)\partial_i \phi$$

[Goldberger, Rothstein (2005)]

[Porto (2007)]

internal unknown DoFs

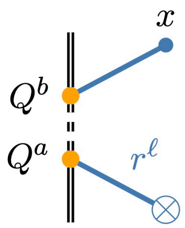
Linear response theory:

$$\phi_{\rm resp} \sim \mathcal{E} r\left(\langle QQ\rangle^{\rm NL}_{\rm ret}\right) \times \left(\frac{r_s}{r}\right)^3 \sim \mathcal{E} r\, \nu\, i r_s(\omega - m\Omega) \times \left(\frac{r_s}{r}\right)^3$$

non-local

Dissipation number

Static dissipation for Kerr due to frame dragging



$$\sim \mathcal{E}_\ell r^\ell \times i r_s(\omega - m\Omega) \times \nu_\ell \left(\frac{r_s}{r}\right)^{2\ell+1}$$

[Chia (2020)]

[Charalambous, Dubovsky, Ivanov (2021)]

[Ivanov, **Zhou** (2022)]

To summarize,

$$S_{\text{finite size}} \supset \int d\tau (Q_E^L(X)E_L + Q_B^L(X)B_L)$$

Love number: time-reversal even part of $\langle QQ\rangle_{\rm ret} \to \lambda_\ell$

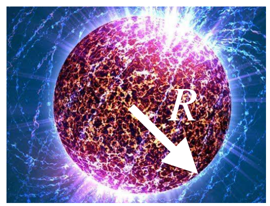
Dissipation number: time-reversal odd part of $\langle QQ\rangle_{\rm ret} \to \nu_\ell$

Love Numbers of Compact Stars

Now, let's turn to spin-2 graviton case:

$$S_{\text{finite size}} = c_E \int d\tau \; E^{ij} E_{ij} + \text{magnetic}$$

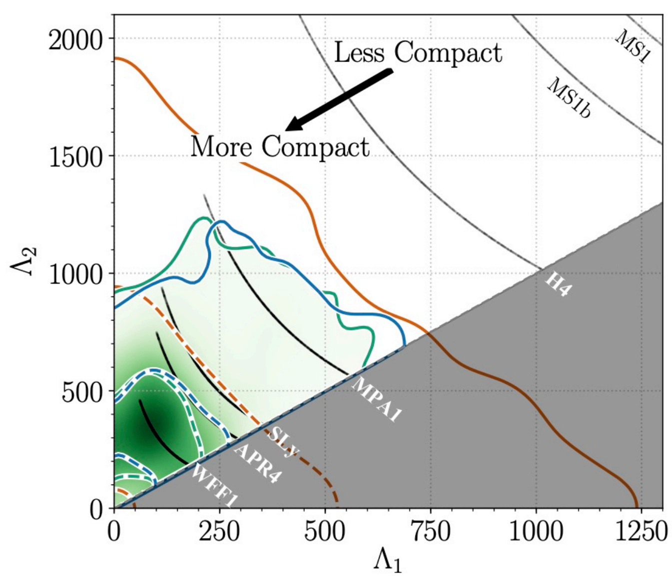
Dimensional analysis



$$c_E \sim R^5$$

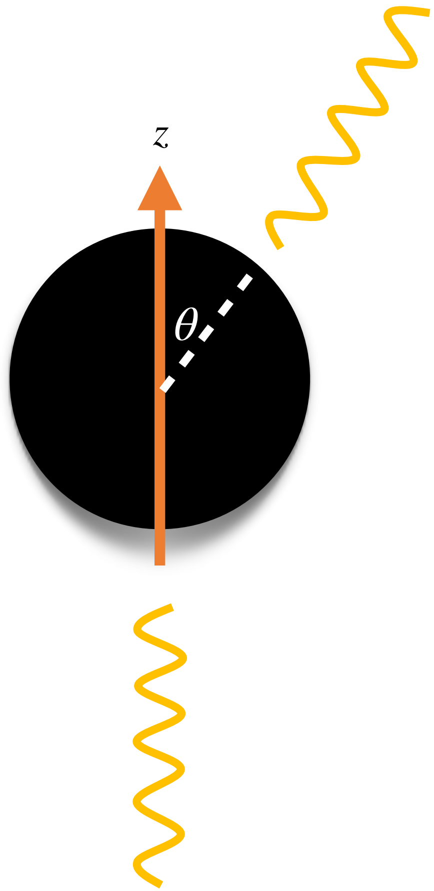
Neutron star:

$$\Lambda \sim R^{-5} c_E$$



[Ligo/Virgo 1805.11581]

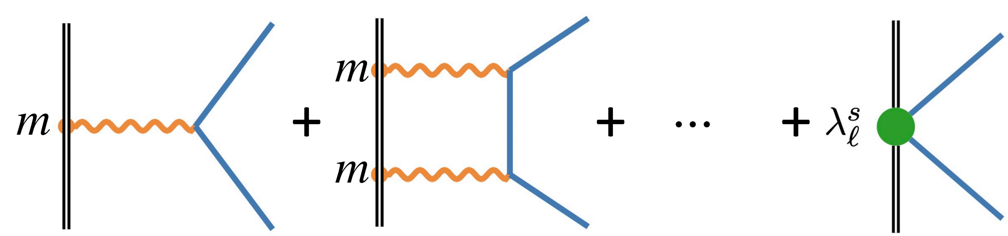
Gauge-invariant matching



$$S_{\text{finite size}} = \int d\tau\, \lambda^{i'j'}_{ij} E^{ij} E_{i'j'} + \text{magnetic}$$

$$\sigma_{\text{Love}}(\omega) \sim \omega^8 \lambda^2 \sim \omega^8 r_s^{10}$$

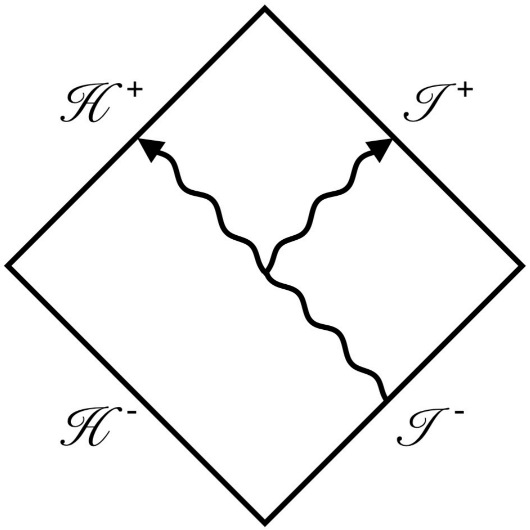
$$\sigma_{\text{GR}}(\omega) = \underbrace{r_s^2}_{\text{Newton}} \underbrace{\left(1 + (r_s\omega)^2 + \ldots\right)}_{\text{rel. corrections}} + \underbrace{r_s^{10}\omega^8}_{\text{finite size}}$$



BH Scattering Formalism & Teukolsky Equation

[Futterman, Handler, Matzner textbook]

[Ivanov, **Zhou**, in progress]



Spin-0: $\phi(\text{plane}) = e^{-i\omega t + i\omega z^*}$

$$\phi(\text{scatt}) = e^{-i\omega t} f_0(\theta) \frac{e^{i\omega r^*}}{r}$$

partial wave scattering amplitude

Spin-1: $A^\mu(\text{plane}) = e^{-i\omega t + i\omega z^*} \epsilon^\mu(\omega \hat{z}, +1) + \text{c.c.}$

$$A^\mu(\text{scatt}) = e^{-i\omega t}\left(f_1(\theta) \frac{e^{i\omega r^*}}{r} \epsilon^\mu(\omega \boldsymbol{k}, +1) + g_1(\theta)^* \frac{e^{i\omega r^*}}{r} \epsilon^\mu(\omega \boldsymbol{k}, -1) \right) + \text{c.c.}$$

helicity-conserving helicity-reversing

Newman-Penrose-Maxwell scalar: $\Phi_2 = F_{\mu\nu} \bar{m}^\mu n^\nu$ spin weight s=-1

Spin-2: $h^{\mu\nu}(\text{plane}) = e^{-i\omega t + i\omega z^*} \epsilon^{\mu\nu}(\omega \hat{z}, +2) + \text{c.c.}$

$$h^{\mu\nu}(\text{scatt}) = e^{-i\omega t}\left(f_2(\theta) \frac{e^{i\omega r^*}}{r} \epsilon^{\mu\nu}(\omega \hat{\boldsymbol{k}}, +2) + g_2(\theta) \frac{e^{i\omega r^*}}{r} \epsilon^{\mu\nu}(\omega \hat{\boldsymbol{k}}, -2) \right) + \text{c.c.}$$

helicity-conserving helicity-reversing

Newman-Penrose-Weyl scalar: $\Psi_4 = - C_{\mu\nu\alpha\beta} n^\mu \bar{m}^\nu n^\alpha \bar{m}^\beta$

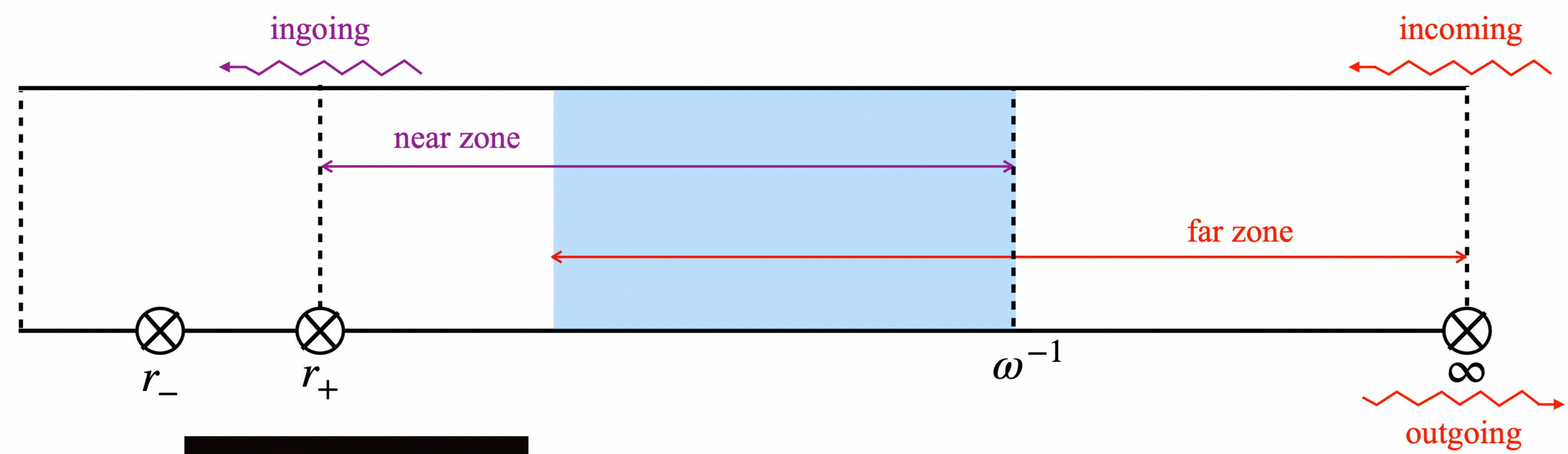
spin weight s=-2

Teukolsky Equation: $\mathscr{T} \psi^{[s]} = 0$ $\quad \psi^{[s]} = e^{im\phi} e^{-i\omega t} {}_sS_\ell^m(\theta; a\omega) {}_sR_{\ell m}(r)$

[Teukolsky (1972)] $\psi^{[0]} = \phi$ $\quad \psi^{[-1]} = \rho^{-2}\Phi_2$ $\quad \psi^{[-2]} = \rho^{-4}\Psi_4$ $\quad$ spin-connection $\rho = (r - ia\cos\theta)^{-1}$

BH Scattering Amplitudes

Teukolsky Equation: $\mathcal{T}\psi^{[s]} = 0$ $\psi^{[s]} = e^{im\phi}e^{-i\omega t}{}_sS_\ell^m(\theta; a\omega)\,{}_sR_{\ell m}(r)$



Near zone: $r\omega \ll 1$

Far zone: $r \gg r_+$

- Build near zone solutions in terms of series of hypergeometric functions
- Build far zone solutions in terms of series of Coulomb wave functions
- Match growing and decaying modes in the intermediate region

[Mano, Suzuki, Takasugi (1996)]

Near Zone/Far Zone Factorization

Phase shift:

$$\eta_{\ell s} e^{2i\delta_{\ell s}} \sim (2\omega)^{-2s} \frac{B^{(\mathrm{refl})}_{-s\ell s}}{B^{(\mathrm{inc})}_{-s\ell s}} \,,$$

Factorization:

$$\frac{B^{(\mathrm{refl})}_{s\ell m}}{B^{(\mathrm{inc})}_{s\ell m}} = \frac{1}{\omega^{2s}} \underbrace{\times \left(1 + 2i(-1)^{\nu} \frac{K_{-\nu-1}}{K_{\nu}} + \cdots \right)}_{\text{Near zone}} \underbrace{\times\, \mathrm{Newton}\,(1 + \cdots)}_{\text{Far zone}}$$

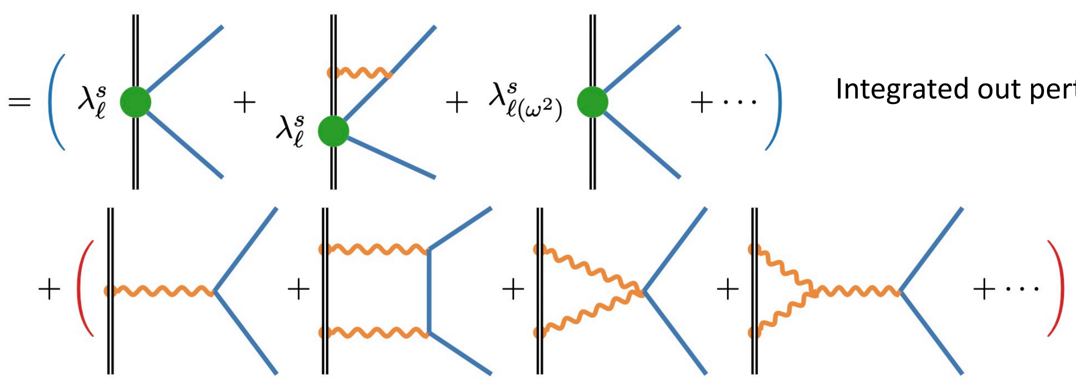
$\frac{K_{-\nu-1}}{K_{\nu}}$: $\frac{\text{decay}}{\text{grow}}$

Newton: $\sim \frac{\Gamma(\nu + 1 - s + 2iM\omega)}{\Gamma(\nu + 1 + s - 2iM\omega)}$

ν : "renormalized" angular momentum $\nu = \ell + \mathcal{O}((M\omega)^2)$

EFT Interpretation

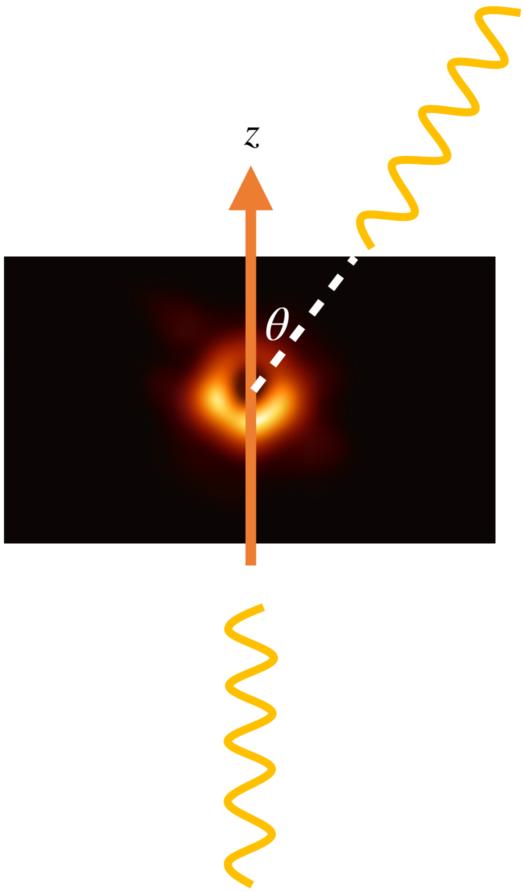
$$\delta_{\ell s} \sim (\text{near zone part}) + (\text{far zone part})$$



Integrated out perturbation potential modes

perturbation wave modes

BH Near Zone Scattering

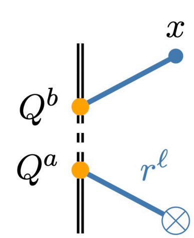


Phase shift: $$\eta_{\ell s} e^{2i\delta_{\ell s}} = (-1)^{\ell+1} \frac{(\ell+s)!}{(\ell-s)!} \frac{1}{(2\omega)^{2s}} \frac{B^{(\mathrm{refl})}_{-s\ell s}}{B^{(\mathrm{inc})}_{-s\ell s}}$$

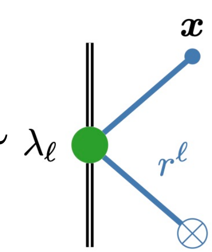
$$\eta_{\ell s} = 1 - (-1)^s \frac{(\ell+s)!(\ell-s)!}{(2\ell)!(2\ell+1)!} \Big(2\tilde{\omega}(\tilde{r}_+ - \tilde{r}_-)\Big)^{2\ell+1} \mathrm{Im}\mathcal{F}_{-s\ell s}$$

$$\delta_{\ell s} = \frac{1}{2}(-1)^s \frac{(\ell+s)!(\ell-s)!}{(2\ell)!(2\ell+1)!} \Big(2\tilde{\omega}(\tilde{r}_+ - \tilde{r}_-)\Big)^{2\ell+1} \mathrm{Re}\mathcal{F}_{-s\ell s}$$

$\mathrm{Im}\mathcal{F}_{-s\ell s} \sim$



$\mathrm{Re}\mathcal{F}_{-s\ell s} \sim \lambda_\ell$



Cross section: $\sigma_{\mathrm{elastic},\ell} = 0$ $\sigma_{\mathrm{abs},\ell} \neq 0$

Higher Dimensional Schwarzschild

- Higher dims: $\hat{d} = d - 3\ ,\quad \hat{\ell} = \dfrac{\ell}{\hat{d}}$

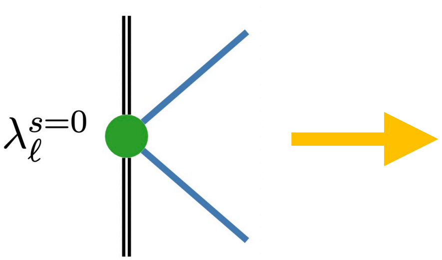
- General $\hat{\ell}$:

$$\eta_\ell = 1 - \frac{2^{1-\hat{d}-2\ell}\pi (r_s\omega)^{\hat{d}+2\ell}}{\Gamma\left(\frac{\hat{d}}{2}+\ell\right)\Gamma\left(1+\frac{\hat{d}}{2}+\ell\right)}\mathrm{Im}\mathcal{I}_\ell \qquad \delta_\ell = \frac{2^{-\hat{d}-2\ell}\pi (r_s\omega)^{\hat{d}+2\ell}}{\Gamma\left(\frac{\hat{d}}{2}+\ell\right)\Gamma\left(1+\frac{\hat{d}}{2}+\ell\right)}\mathrm{Re}\mathcal{I}_\ell$$

$$\mathcal{I}_\ell = \frac{\Gamma(-2\hat{\ell}-1)\Gamma(1+\hat{\ell})\Gamma\left(1+\hat{\ell}-\frac{2ir_s\omega}{\hat{d}}\right)}{\Gamma(-\hat{\ell})\Gamma(2\hat{\ell}+1)\Gamma\left(-\hat{\ell}-\frac{2ir_s\omega}{\hat{d}}\right)}\ .$$

- $\hat{\ell} \notin \mathbb{N}, \mathbb{N}+\frac{1}{2}$ $\quad \mathrm{Re}\mathcal{I}_\ell = \dfrac{2\hat{\ell}+1}{2\pi}\dfrac{\Gamma(\hat{\ell}+1)^4}{\Gamma(2\hat{\ell}+2)^2}\tan(\pi\hat{\ell})$
- $\hat{\ell} \in \mathbb{N}$ $\quad \mathcal{I}_\ell = i\dfrac{r_+\omega}{\hat{d}}\dfrac{(\ell!)^2}{(2\ell)!(2\ell+1)!}\displaystyle\prod_{j=1}^{\ell}\left(j^2+4\left(\frac{r_s\omega}{\hat{d}}\right)^2\right)$
- $\hat{\ell} \in \mathbb{N}+\frac{1}{2}$ $\quad \mathcal{I}_\ell$ diverge

- Love number:



$$\lambda_\ell^{s=0} \sim r_s^{2\ell+d-3}\mathrm{Re}\mathcal{I}_\ell \qquad r_s^{2\ell+d-3} \sim (GM)^{(2\hat{\ell}+1)}$$

- $\hat{\ell} \notin \mathbb{N}, \mathbb{N}+\frac{1}{2}$ — no mixing with EFT Loop
- $\hat{\ell} \in \mathbb{N}$ — mix with EFT Loop but vanish
- $\hat{\ell} \in \mathbb{N}+\frac{1}{2}$ — mix with EFT Loop and has RG running

Classical RG Running

- Angular momentum reg: $\hat{\ell} = \frac{n}{2} - \frac{\epsilon}{2}$

$$(r_s\omega)^{-\hat{d}\epsilon}\mathrm{Re}\mathcal{I}_\ell = (-1)^{2\hat{\ell}+1}\frac{\Gamma(1+\hat{\ell})^2}{\Gamma(-\hat{\ell})^2}\frac{1}{(2\hat{\ell})!(2\hat{\ell}+1)!}\left(\frac{1}{\epsilon} - \hat{d}\log(r_s\omega)\right)$$

ignore divergent term, can be absorbed into local c.t.

- Logs in cross section:

$$\sigma_{\mathrm{elastic},\ell} \sim r_s^{4\ell+2\hat{d}}\omega^{4\ell+\hat{d}-1} \times \log^2(\omega r_s) \sim \omega^{4\ell+\hat{d}-1}\left((GM)^{2\hat{\ell}+1}\log(\omega r_s)\right)^2$$

even number

- Introduce sliding scale μ:

$$\log(r_s\omega) = \underbrace{\log\left(r_s\mu\right)}_{\text{Love number}} + \underbrace{\log\left(\frac{\omega}{\mu}\right)}_{\text{EFT loops}}$$

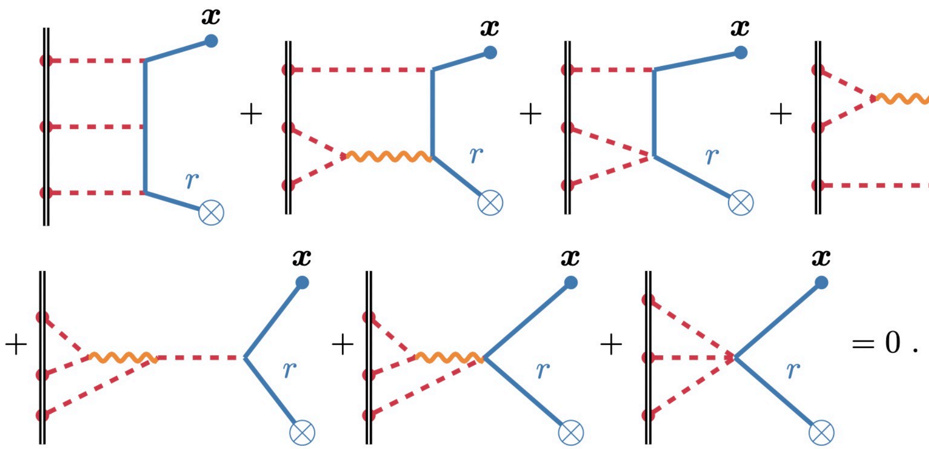
Wilsonian Naturalness

- Expect Love number $\lambda_\ell \sim \mathcal{O}(1)$

- But for all 4D BH $\lambda_\ell = 0$

- Expect Classical RG running

- But the summation of all loops cancel



[Ivanov, **Zhou** (2022)]

“Hidden” Love Symmetry

$$L_0 = -\beta\partial_t \qquad \beta = (2\pi T_H)^{-1}, \quad \Delta = (r - r_+)(r - r_-)$$

[Charalambous, Dubovsky, Ivanov (2021)]

$$L_{\pm1} = e^{\pm\beta^{-1}t}\left(\mp\Delta^{1/2}\partial_r + \beta\partial_r\left(\Delta^{1/2}\right)\partial_t + \frac{a}{\Delta^{1/2}}\partial_\phi\right)$$

- Regular at the horizon

- Satisfy $\mathrm{SL}(2,\mathbb{R})$ algebra

$$[L_n, L_m] = (n-m)L_{n+m}, \quad n, m = -1,0,1$$

- Near zone Teukolsky equation

$$\mathscr{C}_2\phi = \ell(\ell+1)\phi \qquad \text{Casimir} \quad \mathscr{C}_2 \equiv L_0^2 - \frac{1}{2}(L_{-1}L_1 + L_1L_{-1})$$

- Static solution follows highest weight rep of $\mathrm{SL}(2,\mathbb{R})$

$$L_0\phi = h\phi, \quad h = 0$$ → polynomial in radial direction, no response

Subtracted Geometry & Ladder Symmetry

Subtracted geometry: effective geometry to realize near zone Teukolsky

$$ds^2_{\text{near-zone}} = -\frac{\Delta}{r_s^2}\, dt^2 + \frac{r_s^2}{\Delta} dr^2 + r_s^2\, d\Omega_S^2$$

[Cvetic, Larsen (2011)]

[Cvetic, Larsen (2012)]

[Hui et al (2022)]

→ $\mathrm{AdS}_2 \times S^2$

→ Killing vectors of subtracted geometry give the $\mathrm{SL}(2,\mathbb{R})$ generators of Love symmetry

→ Vanishing Love number as a algebraic constraint of highest weight rep

"Ladder" symmetry? [Hui et al (2021)] [Hui et al (2022)]

→ Conformal Killing vectors give the generator of "Ladder" symmetry

→ Create unphysical state ($|m| > \ell$)

→ Mix different multipoles ℓ

Summary

- ☑ Worldline EFT— gauge invariant def of Love and dissipation

- ☑ BH near zone scattering—gauge invariant way of computing Love numbers

- ☑ Higher dimensional BH—classical RG running

- ☑ 4D BH—vanishing Love numbers due to Love symmetry